CAREER: Algebraic Knowledge for Teaching: A Cross-cultural Perspective (2014-19)

Meixia Ding (Principle Investigator), **Ying Yu** (Graduate student), **Temple University**

Overview of the Project

This five-year project (2014-2019) aims to identify, from a cross-cultural perspective, essential *algebraic knowledge for teaching* (AKT) that fosters students' algebraic thinking in elementary school. Focusing on two fundamental mathematical ideas that are early algebra topics – *inverse relations* and *properties of operations* (Common Core State Standards Initiative, 2010) – this study explores AKT based on integrated insights of U.S. and Chinese expert teachers' classroom performance. This study is innovative because it is among the very first to seek AKT focusing on fundamental mathematical ideas from a cross-cultural perspective. The conceptual framework for identifying AKT is aligned with high-quality cognitive research recommendations on worked examples, representations, and deep questions (Pashler et al., 2007). It is expected that the identified AKT along with these aspects will contribute to students' deep understanding of fundamental mathematical ideas and thus algebraic readiness.

Objectives of the Project

Objective #1: Identify AKT that facilitates algebraic thinking and develop preliminary findings into teaching materials (Ys1-3).

Objective #2: Disseminate preliminary findings and refine research-based teaching materials based on evaluative data (Ys3-4)

Objective #3: Integrate research with education through course development at Temple and teacher outreach in Philadelphia (Ys3-5)

	Research	Education
Year 1	Data collection: Inverse Relations	
Year 2	Data collection: Properties of Operations	
Year 3	Data analysis & material development Workshops for teachers in both countries	US-China online teacher forum
Year 4	Reteach lessons & material refinement	US-China video conference Chinese classroom visit Workshops for SDP novice teachers
Year 5		SDP teacher conference Temple course revision and development

Participants

US teachers: Philadelphia; Chinese teachers: Nanjing

Y	Topic	US Classroom		Chinese Classroom		# of Lessons/Teacher	# of taped lessons
		# of teachers	# of students	# of teachers	# of students		
1	Inverse Relations	8	200	8	400	4	64
2	Properties of Operations	9	225	8	450	4	68
4	Both topics	12	300	12	600	4	96[1]

[1]One US teacher taught 2 lessons.

Materials and Instruments

Each teacher taught 4 lessons on either inverse relations or the basic properties of operations based on teachers' existing textbooks. In years 1-2, the US textbooks included *Investigations*, *Go Math, and My Math*. However, in year 4, the school district adopted two new textbook series: *enVisionmath2.0* and *Math Expressions*. *Investigations* remained in some classrooms. The Chinese textbook series was *Jiang Su Educational Press* (JSEP), which was not changed.

US / China

Y1-2

Y4

To link AKT to student learning gains, we conducted student pre- and post-tests on inverse relations (Y1) and the basic properties of operations (Y2).

Coding

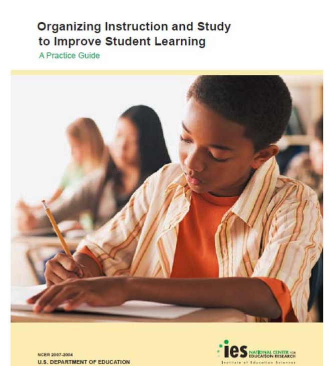
Videos: A 0–2 scale was used to rate each lesson in terms of three dimensions aligned with the IES recommendations (Pashler et al., 2007), which were detailed into six sub-categories:

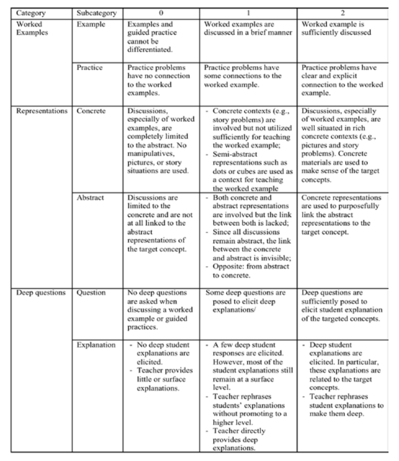
Worked examples: Interleaving worked examples with problem solving exercises (example + practice)

Representations: Making connections between concrete and abstract representations (concrete + abstract)

Deep questions: Asking deep questions to elicit student self-explanations (question + explanation)

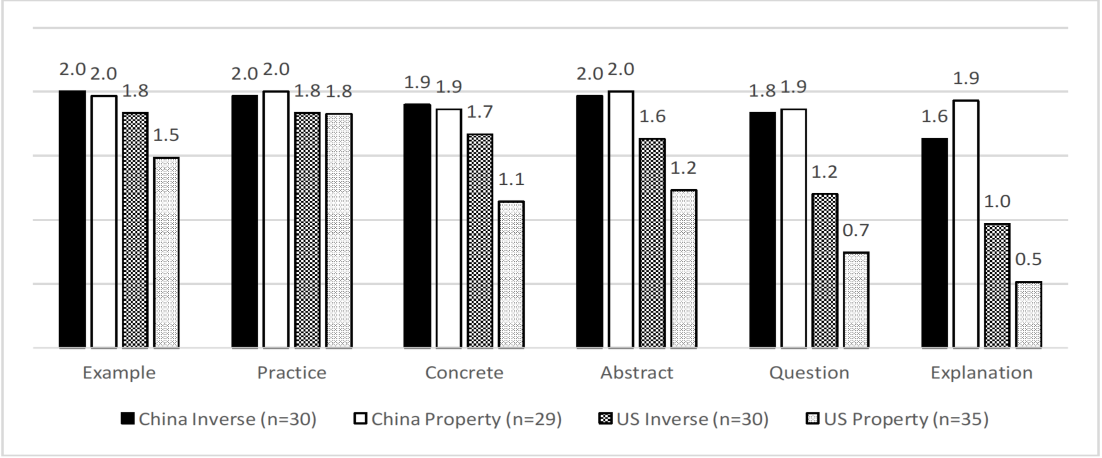
Student Tests: The pre- and post- tests were also rated for both math topics.





Results: Identifying AKT (Ys1-3)

A video analysis of 132 US and Chinese lessons on inverse relations and the properties of operations indicates instructional differences. Below shows the mean score differences (out of 2):

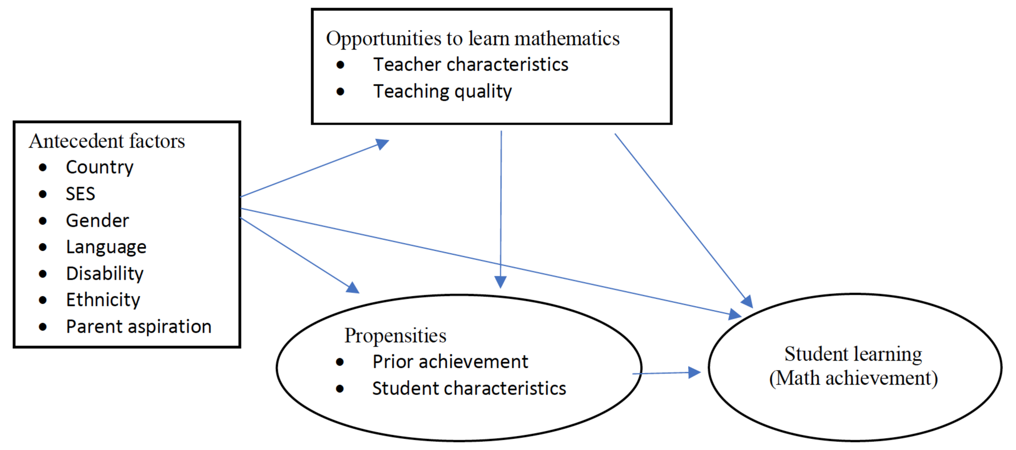


Based on quantitative and qualitative analyses, we concluded with three AKT components:

Worked example: Unpacking one worked example sufficiently to illustrate the targeted concept/ big idea. It is not effective to teach many repetitive examples without making the big idea explicit.
Representation: Situating the new teaching in a real-word context which should be gradually faded out into abstract. It is not effective to use concrete representations to find computational answers. Rather, these tools should be used to illustrate quantitative relationships (Ding, in press; Ding, Chen, & Hassler, 2018).
Deep question: Asking deep follow-up questions that targeted the meaning of operations and quantitative relationships. One types of deep questions is comparison with follow-ups.
Illustration of these results include 25 annotated video clips that contain merits of various aspects.

Partial Results on the Impact of AKT (Y1)

Using the Y1 teaching and learning data on inverse relations, we examined whether instruction aligned with IES recommendations predicts student learning of early algebra in elementary classrooms. Instructional quality was determined in an opportunity-propensity analysis（Ding, Byrnes, Barnett, and Hassler, 2018).



Results indicates that teaching plays a stronger role in student learning (N = 589) than previously reported. The full model explained a total of 58.4% of the variance. After controlling for the covariates of antecedent (e.g., SES) and propensity factors (e.g., prior achievement) as well as the teacher characteristics (e.g., self-efficacy), teaching quality - especially teachers' use of representations and deep questions – explains additional variance (6.6%), which is significant. The pattern held in both the US and China data even though there were several interesting differences in responses. This indicates that despite cross-cultural differences in the predictability of O-P model, the factor of "teaching quality" in alignment with the IES recommendations (worked examples, representations, and deep questions) consistently plays a significant role in predicting students' early algebra learning across both countries in the sampled data sets.

Results of Disseminating/Refining AKT (Ys3-4)

To disseminate and refine our findings, we conducted an one-month US-China online video-forum through the platforms of Youtube (US) and YouKu (China). Teachers in both countries watched and commented 25 annotated video clips with pedagogical merits, followed by 20-hour summer workshops. This serves as a project intervention, which led to teachers' re-teaching of the targeted lessons in Y4 (Ding Manfredonia, & Luo, 2018).

An analysis of teachers' video comments indicates:

- Teachers in both countries showed great interests and learning desire from watching international peers' videos
- *US Teachers* were particularly impressed by the depth of mathematics that children engaged in Chinese classrooms and teachers' instructional approaches (e.g., concreteness fading, deep follow-up questions) to pursue such depth
- *Chinese Teachers* were more interested in US relaxed classroom climate and teachers' use of concrete representations to enable students to naturally explore ideas. They reflected why Chinese classrooms seemed to lack such atmosphere and whether the type of mathematical depth pursued in Chinese classrooms was necessary for students' well-rounded growth.

These findings inform us that our identified AKT is feasible for practice. We are currently analyze Y4 videos to understand how teachers actually transform what they've learned to classrooms. We are also in the process of disseminating the findings to more teachers in the school districts through PD workshops (Ys4-5).

For more information, please watch NSF2018 video showcase:
http://stemforall2018.videohall.com/presentations/1143

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