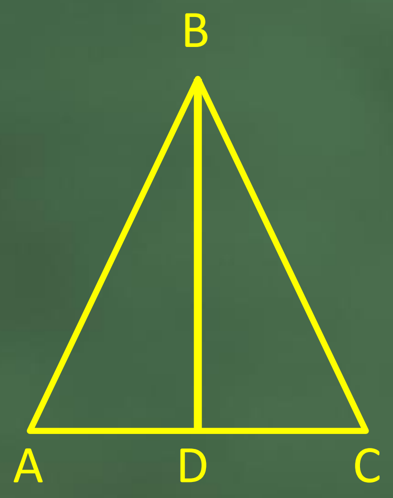
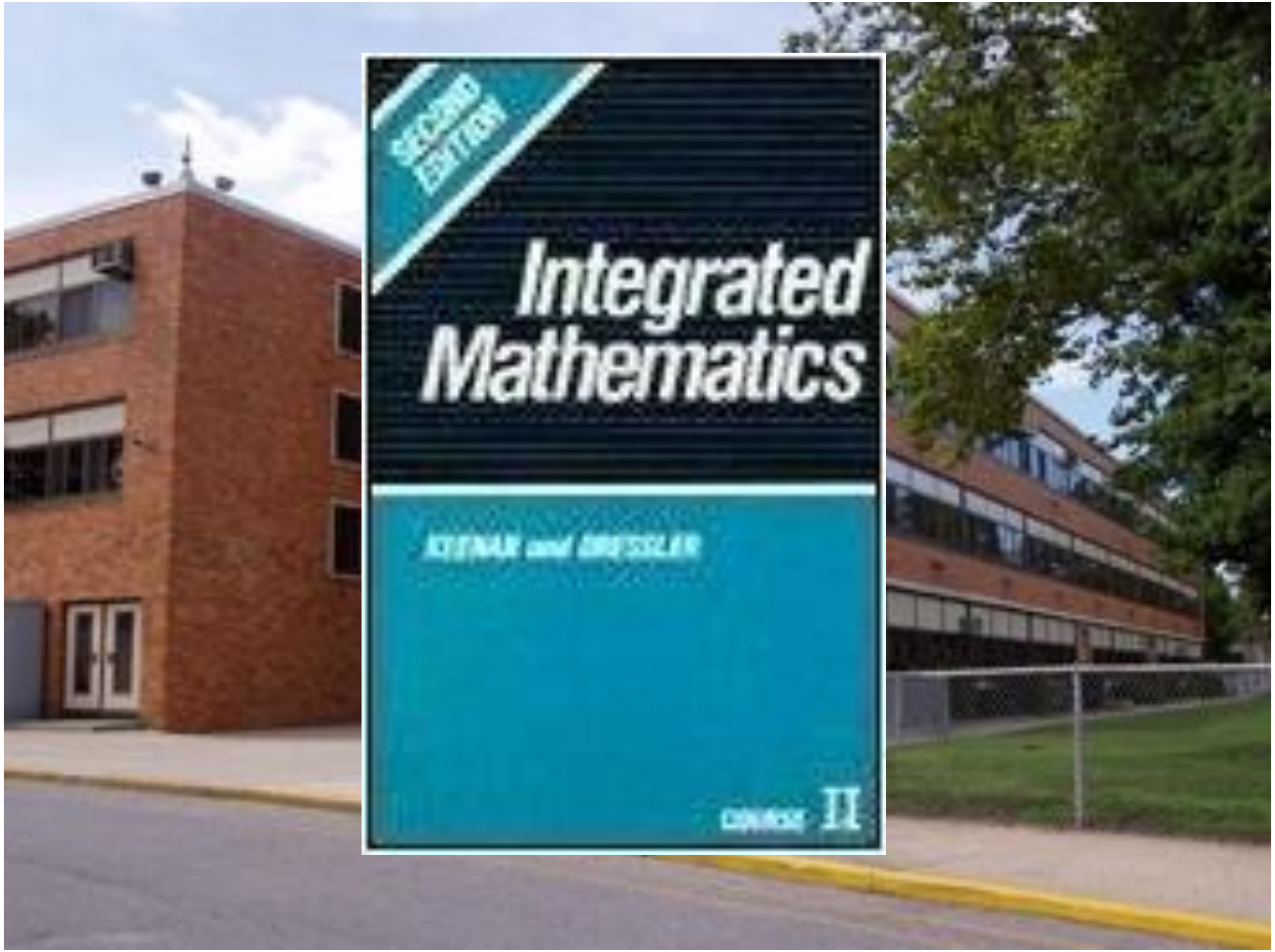
Building Mathematical Knowledge for Teaching Proof in Geometry

Michelle Cirillo
Department of Mathematical Sciences
University of Delaware, USA
TWITTER: @UDMichy



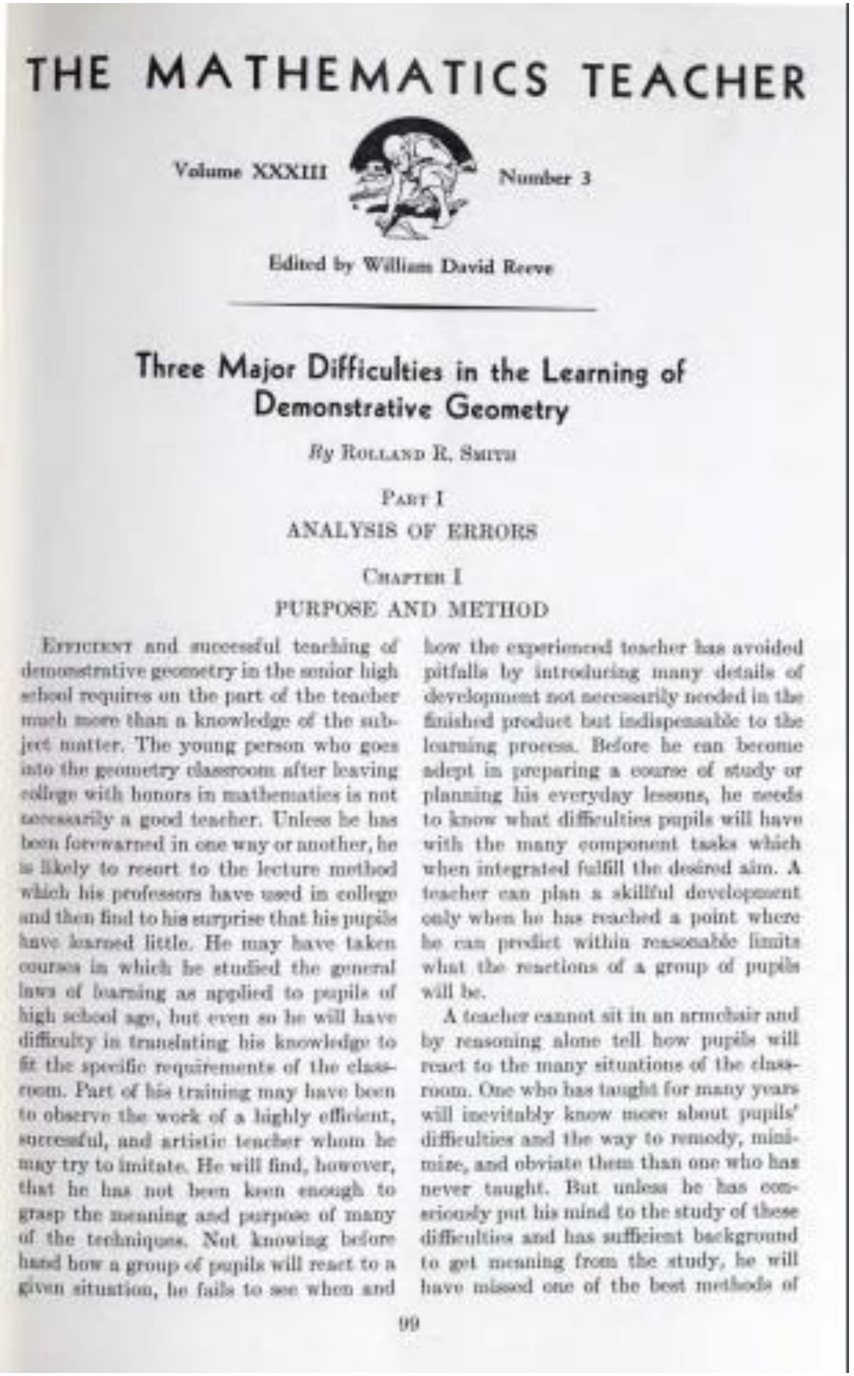
Presented at:
NCTM Nashville Regional
October 3, 2019

What makes teaching proof difficult?


SECOND EDITION
Integrated Mathematics
COURSE II

Three Major Difficulties in the Learning of Demonstrative Geometry

Rolland R. Smith (1940)



THE MATHEMATICS TEACHER

Volume XXXIII Number 3

Edited by William David Reeve

Three Major Difficulties in the Learning of Demonstrative Geometry

By Rolland R. Smith

Part I
ANALYSIS OF ERRORS

Chapter I
PURPOSE AND METHOD

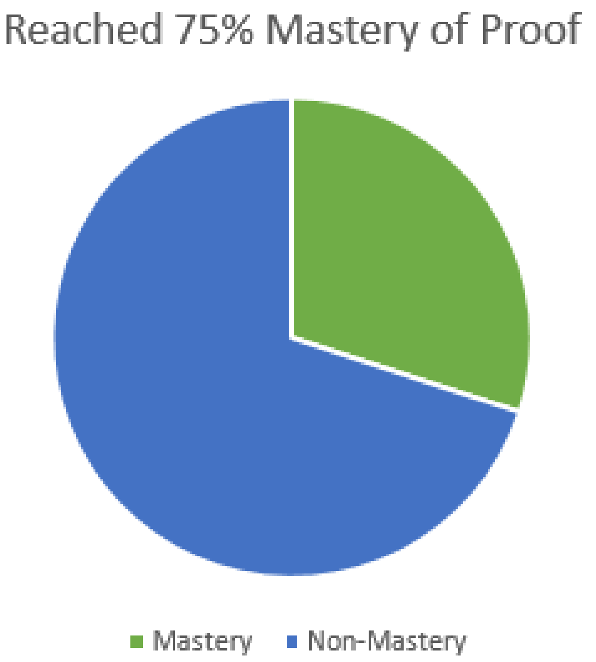
Efficient and successful teaching of demonstrative geometry in the senior high school requires on the part of the teacher much more than a knowledge of the subject matter. The young person who goes into the geometry classroom after leaving college with honors in mathematics is not necessarily a good teacher. Unless he has been forewarned in one way or another, he is likely to resort to the lecture method which his professors have used in college and then find to his surprise that his pupils have learned little. He may have taken courses in which he studied the general laws of learning as applied to pupils of high school age, but even so he will have difficulty in translating his knowledge to fit the specific requirements of the classroom. Part of his training may have been to observe the work of a highly efficient, successful, and artistic teacher whom he may try to imitate. He will find, however, that he has not been keen enough to grasp the meaning and purpose of many of the techniques. Not knowing beforehand how a group of pupils will react to a given situation, he fails to see when and how the experienced teacher has avoided pitfalls by introducing many details of development not necessarily needed in the finished product but indispensable to the learning process. Before he can become adept in preparing a course of study or planning his everyday lessons, he needs to know what difficulties pupils will have with the many component tasks which when integrated fulfill the desired aim. A teacher can plan a skillful development only when he has reached a point where he can predict within reasonable limits what the reactions of a group of pupils will be.

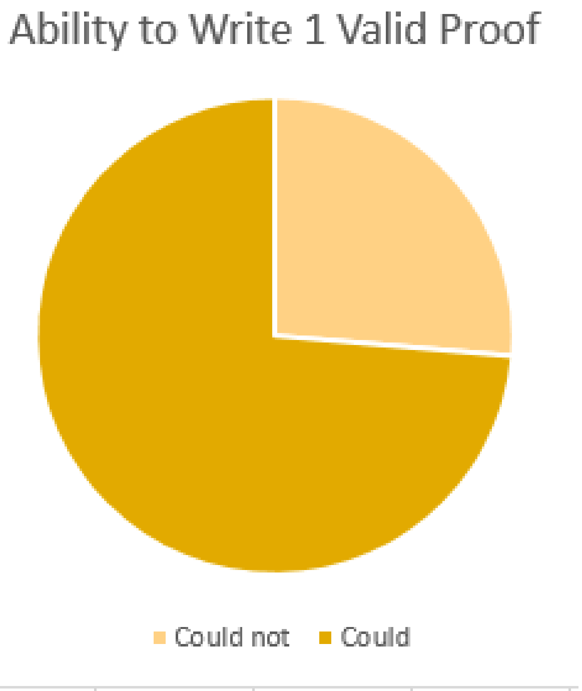
A teacher cannot sit in an armchair and by reasoning alone tell how pupils will react to the many situations of the classroom. One who has taught for many years will inevitably know more about pupils' difficulties and the way to remedy, minimize, and obviate them than one who has never taught. But unless he has consciously put his mind to the study of these difficulties and has sufficient background to get meaning from the study, he will have missed one of the best methods of

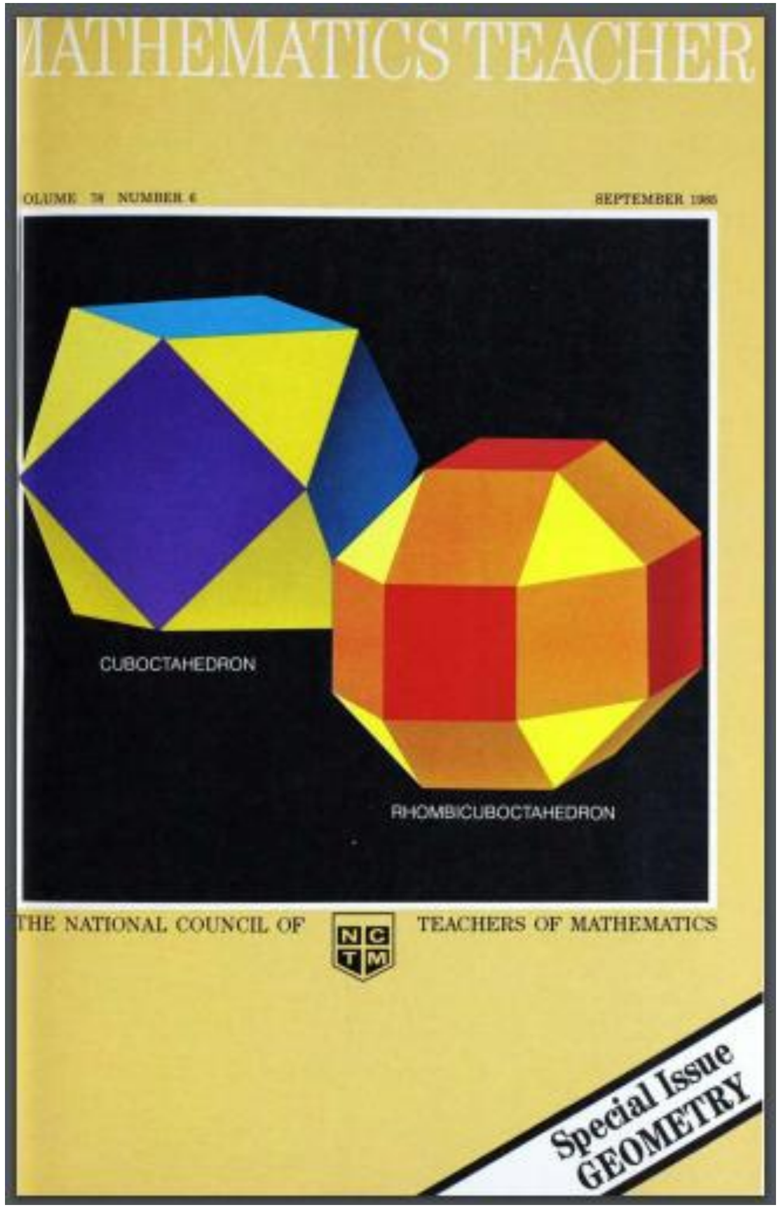
“Three Serious Learning Difficulties”

- Lack of familiarity with <u>geometric figures</u>
- Not sensing the meaning of the <u>if-then</u> relationship
- Inadequate understanding of the <u>meaning of proof</u>

How well do students write geometry proofs?







Sharon Senk (1985)

Senk's Recommendations

We must immediately look for more effective ways to teach proof in geometry. We should:

- Pay special attention to teaching students to *start* a chain of reasoning;
- Place greater emphasis on the meaning of proof than we do currently; and
- Teach students how, why, and when they can transform a diagram in a proof.

Research on Proof in School Geometry

- Proof is important – the “guts of mathematics” (Wu, 1996).

BUT

- Proof is challenging for teachers to teach (e.g., Knuth, 2002, Cirillo, 2009; 2014).

- Proof is difficult for students to learn (Senk, 1985; McCrone & Martin, 2004).

Students’ Difficulties

- “In summary, we have seen that students are extremely unsuccessful with formal proof in geometry.”

(Clements & Battista, 1992)

- “The teaching of mathematical proof appears to be a failure in almost all countries.”

(Hershkowitz et al., 2002, p. 675)

Three Studies

- 2005-2008: Longitudinal Dissertation Study
- 2010-2013: The Geometry Proof Project
- 2015-2020: Proof in Secondary Classrooms:
  Decomposing a Central Mathematical Practice
  (i.e., The PISC Project)

Study 1: The Case of Matt

Matt

You can't teach somebody how to do a proof....I mean if a student's really gonna do a mathematical proof, you look at the problem and you either see how you do it or you don't.

Matt

I mean you don't want to go so far as to say it doesn't matter what I do, but the reality is that I can't prove it for them. You know, simply showing somebody how to do a proof will help, but only up to a certain point. Only until they understand...the way in which a proof becomes a proof.

Matt

I'm like a Sherpa. Okay? That's the word I'm looking for. So...you know, I've been up and down the mountain 50 times....Yeah, I'm like the Sherpa guide who just walks with you up the mountain, but then at base camp I just, I go off and meditate somewhere else and I really don't pay attention to what you're doing. And I don't just have one person - I'm trying to herd like 30 people to the top of the mountain before next Friday.

Textbook Examples

- Reasoning with Properties from Algebra

EXAMPLE 2 ***Writing Reasons***

Solve $55z - 3(9z + 12) = -64$ and write a reason for each step.

SOLUTION

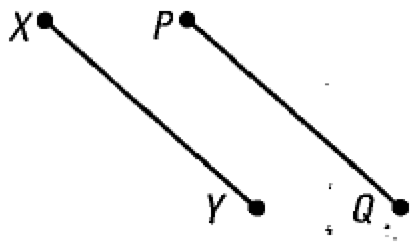
$55z - 3(9z + 12) = -64$	Given
$55z - 27z - 36 = -64$	Distributive property
$28z - 36 = -64$	Simplify.
$28z = -28$	Addition property of equality
$z = -1$	Division property of equality

Textbook Examples

- Proving Statements about Segments

EXAMPLE 1 ***Symmetric Property of Segment Congruence***

You can prove the Symmetric Property of Segment Congruence as follows.



GIVEN ▸ $\overline{PQ} \cong \overline{XY}$

PROVE ▸ $\overline{XY} \cong \overline{PQ}$

Statements	Reasons
1. $\overline{PQ} \cong \overline{XY}$	**1.** Given
2. $PQ = XY$	**2.** Definition of congruent segments
3. $XY = PQ$	**3.** Symmetric property of equality
4. $\overline{XY} \cong \overline{PQ}$	**4.** Definition of congruent segments

Overall Findings

- Despite strong content knowledge and a good teacher prep program, Matt was at a loss for teaching proof beyond show-and-tell.
- Matt wanted to teach "real math," not just show students completed Theorems in the boxes in his textbook.
- Matt's focus shifted from getting through the required theorems to attempting to teach students to prove.

Study 2: The Case of Mike

Mike, High School Geometry Teacher

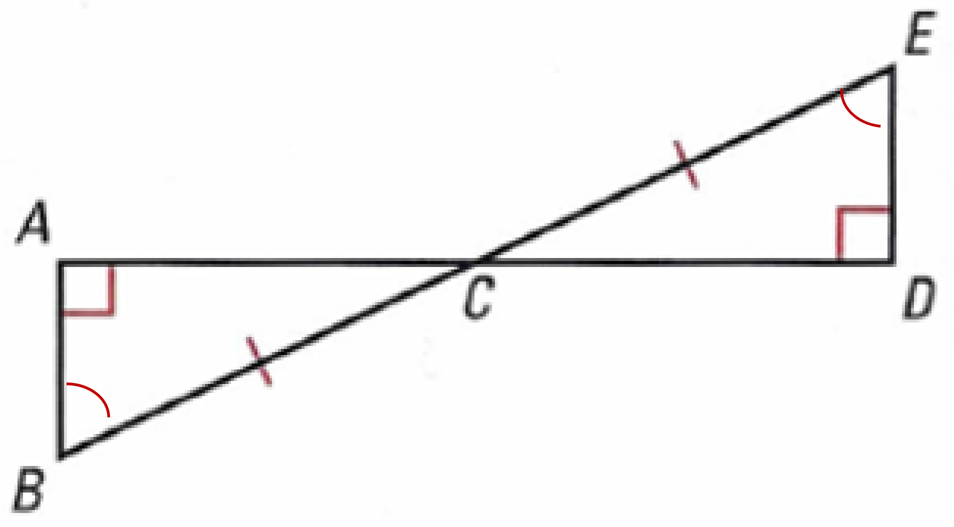
- 8 years of experience at start of project
- Mathematics and Science background
- Conventional Prentice Hall *Geometry* textbook
- Private boys' school
- Described students as motivated, curious, confident, intelligent, and affluent

Mike Began Proof with Triangle Congruence

1. GIVEN: $\angle A \cong \angle D$, $\angle B \cong \angle E$

$\overline{BC} \cong \overline{EC}$

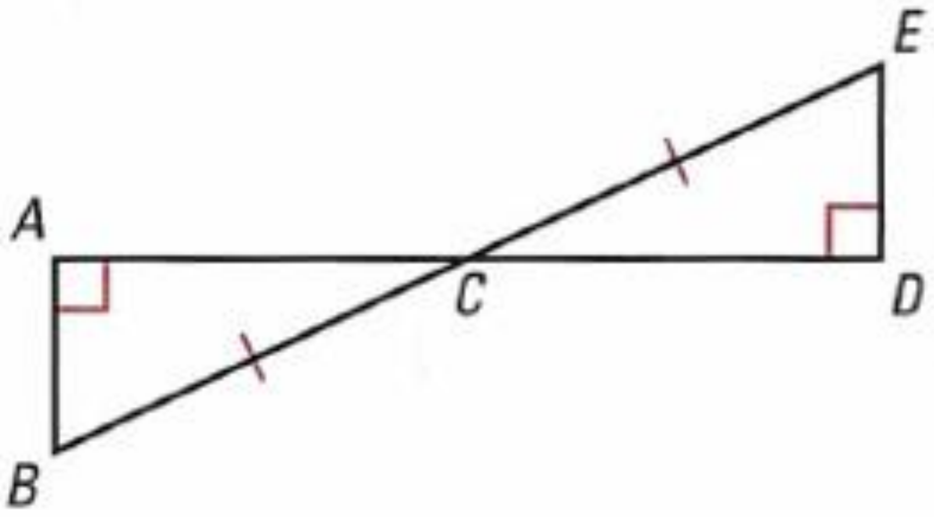
PROVE: $\Delta ABC \cong \Delta DEC$



20. GIVEN ▶ $\overline{AB} \perp \overline{AD}$, $\overline{DE} \perp \overline{AD}$,

$\overline{BC} \cong \overline{EC}$

PROVE ▶ $\triangle ABC \cong \triangle DEC$



Mike, Year 1, Day 1

- VIDEO REMOVED

Back to Matt for a Brief Moment…

Matt – Year 2

- “On Friday the students will begin constructing their own deductive proofs. Unfortunately, there is no good way, in my opinion, to ‘teach’ proofs. Students simply have to do them – like learning to swim by drowning.”

- “Ok, there's only so many of these that I can do with us together. I just kind of, got to keep throwing you in the deep end. Letting you thrash around for awhile. And then throw you a floaty. Haul you back out and then throw you back in. Alright?”

(Cirillo, 2008)

Back to Mike…

Things I need to know:

- How do I know what steps to write?
- How do I know what order the steps are in?
- Argh! I don’t even know where to start!!!
- How big should I make the T?
- What reasons am I allowed to use?
- How many steps do I need to write?

Mike, Year 1, Day 2

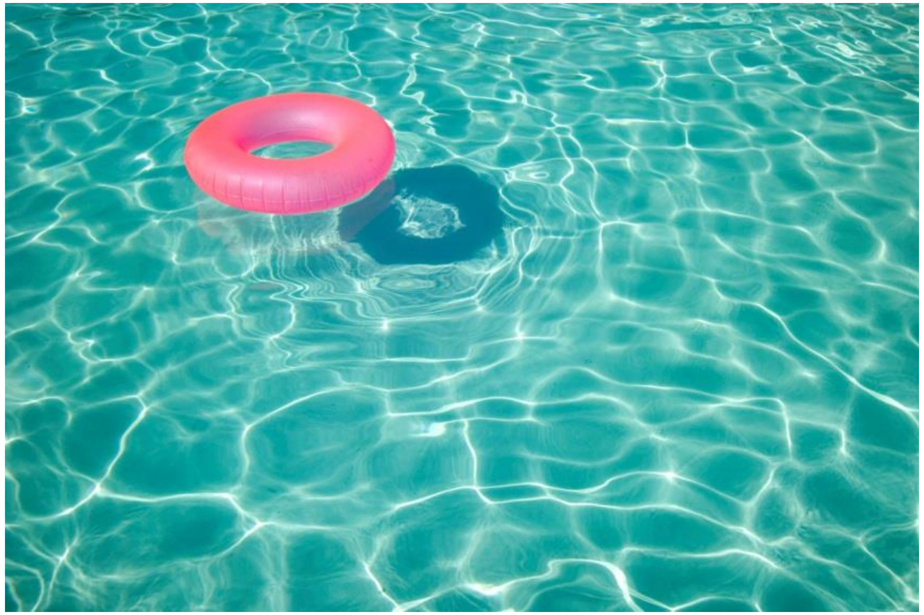
- VIDEO REMOVED

What makes teaching proof in geometry so tough?

- Curriculum
- Student Readiness
- Lack of recommendations for scaffolding the introduction to proof (i.e., understanding of the “shallow end” of the proof pool)

So what’s a geometry teacher to do?

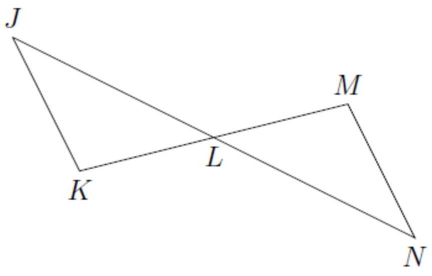
- What is going on for students when we introduce proof?



A Proof

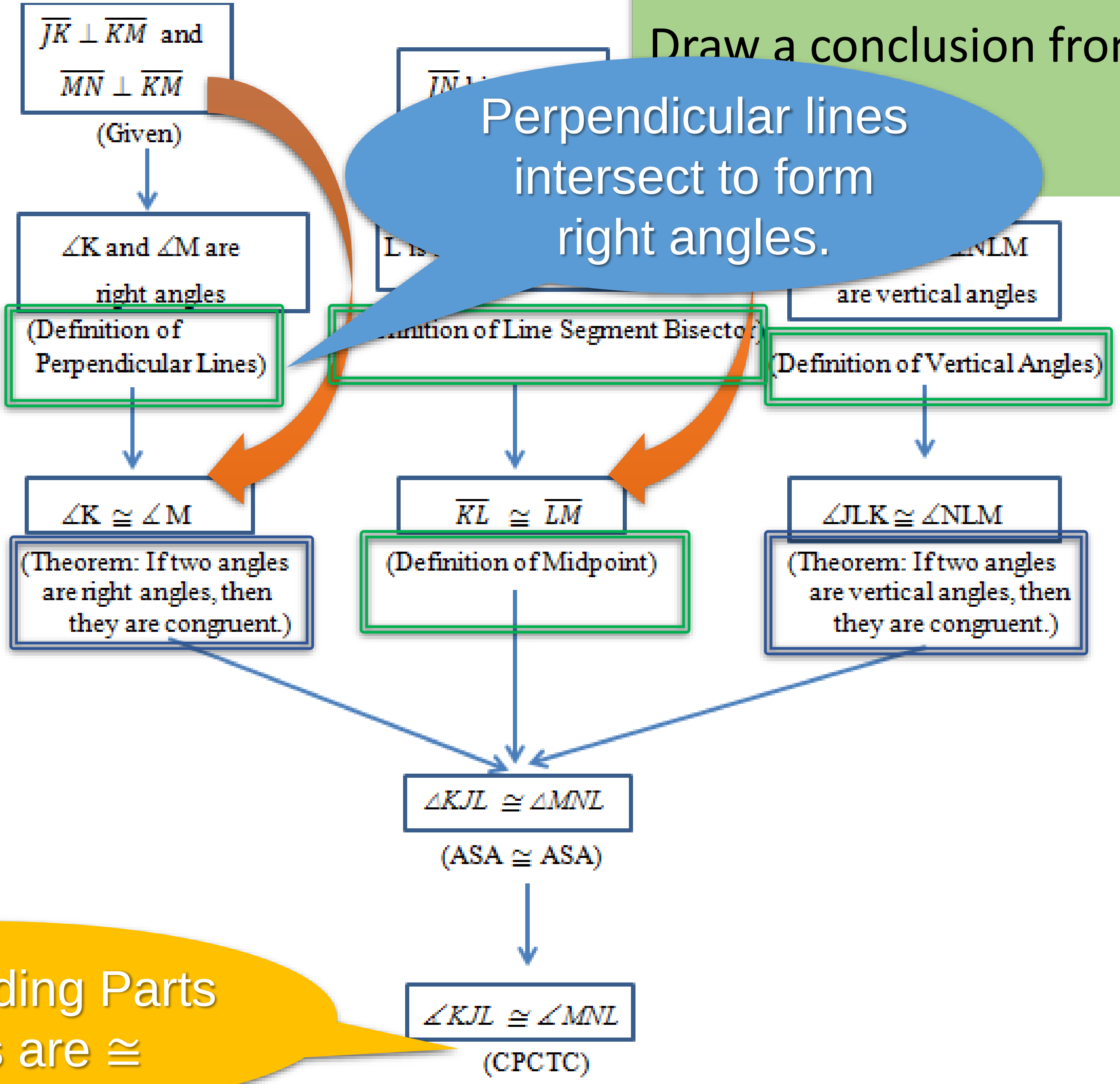
Given: $\overline{JN}$ bisects $\overline{KM}$
$\overline{JK} \perp \overline{KM}$
$\overline{MN} \perp \overline{KM}$

Prove: $\angle KJL \cong \angle MNL$



Statements	Reasons

No Given?
Draw a conclusion from



Perpendicular lines intersect to form right angles.

Corresponding Parts of ≅ △s are ≅

There is much to learn about "simple" proofs…

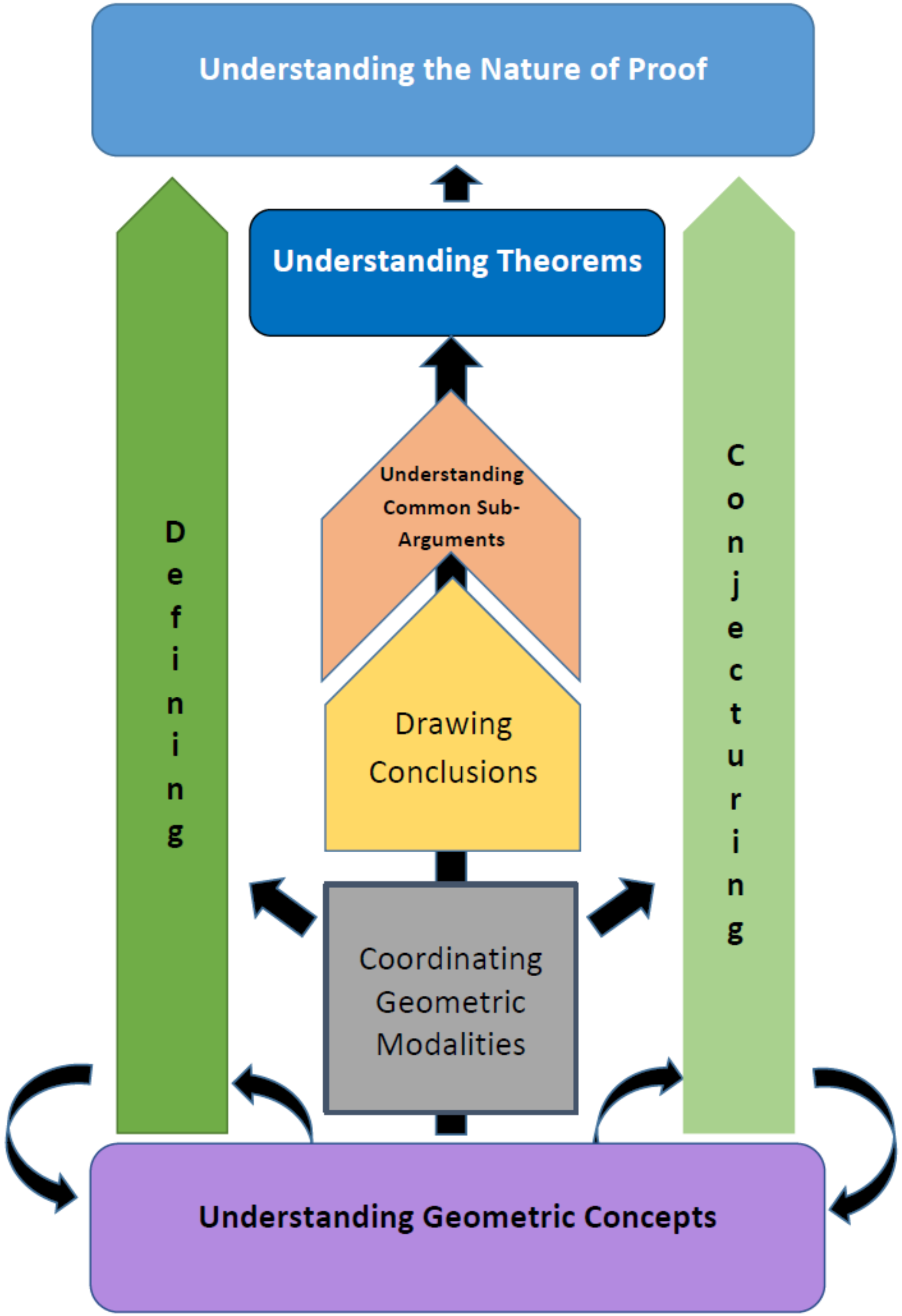
- particular postulates, definitions, and theorems;
- how to *use* definitions and theorems to draw conclusions
- how to work with diagrams;
- a variety of sub-arguments and classroom norms for writing them up; and
- how sub-arguments come together to construct a larger argument.

If there was a shallow end to teaching proof, what would it look like?

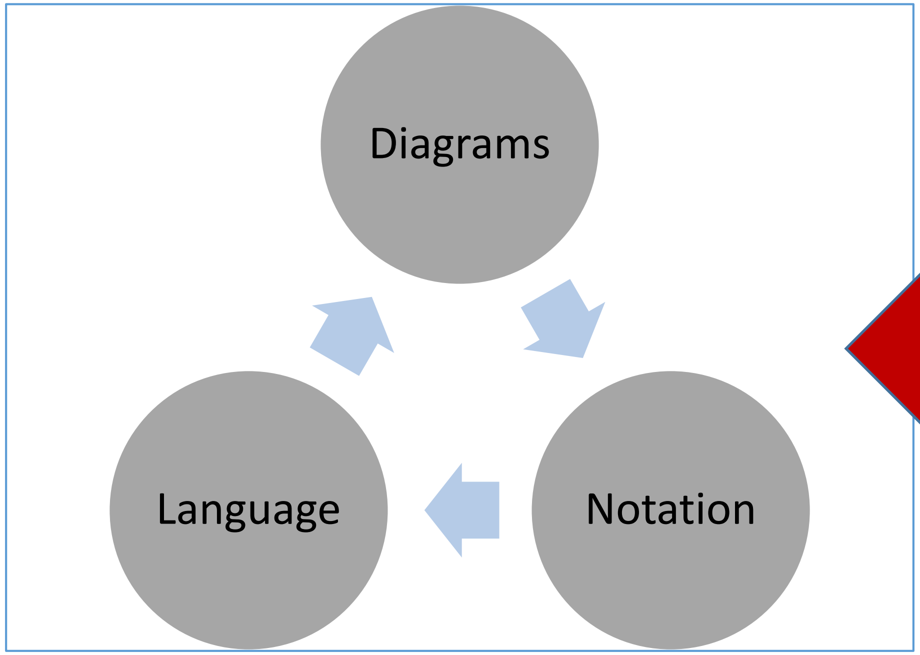
Study 3: The PISC Project

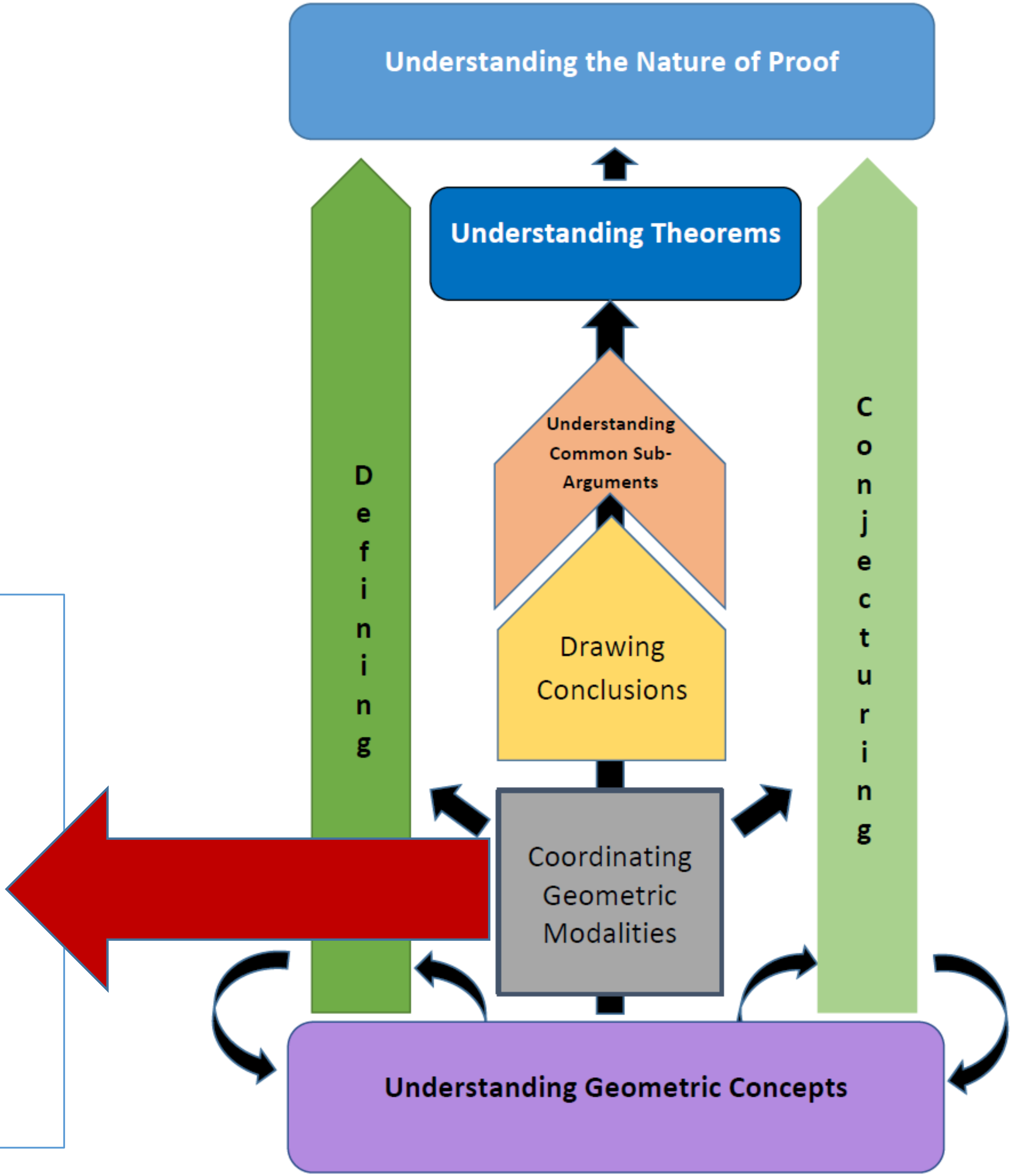
The Geometry Proof Scaffold

(i.e., the "GPS")



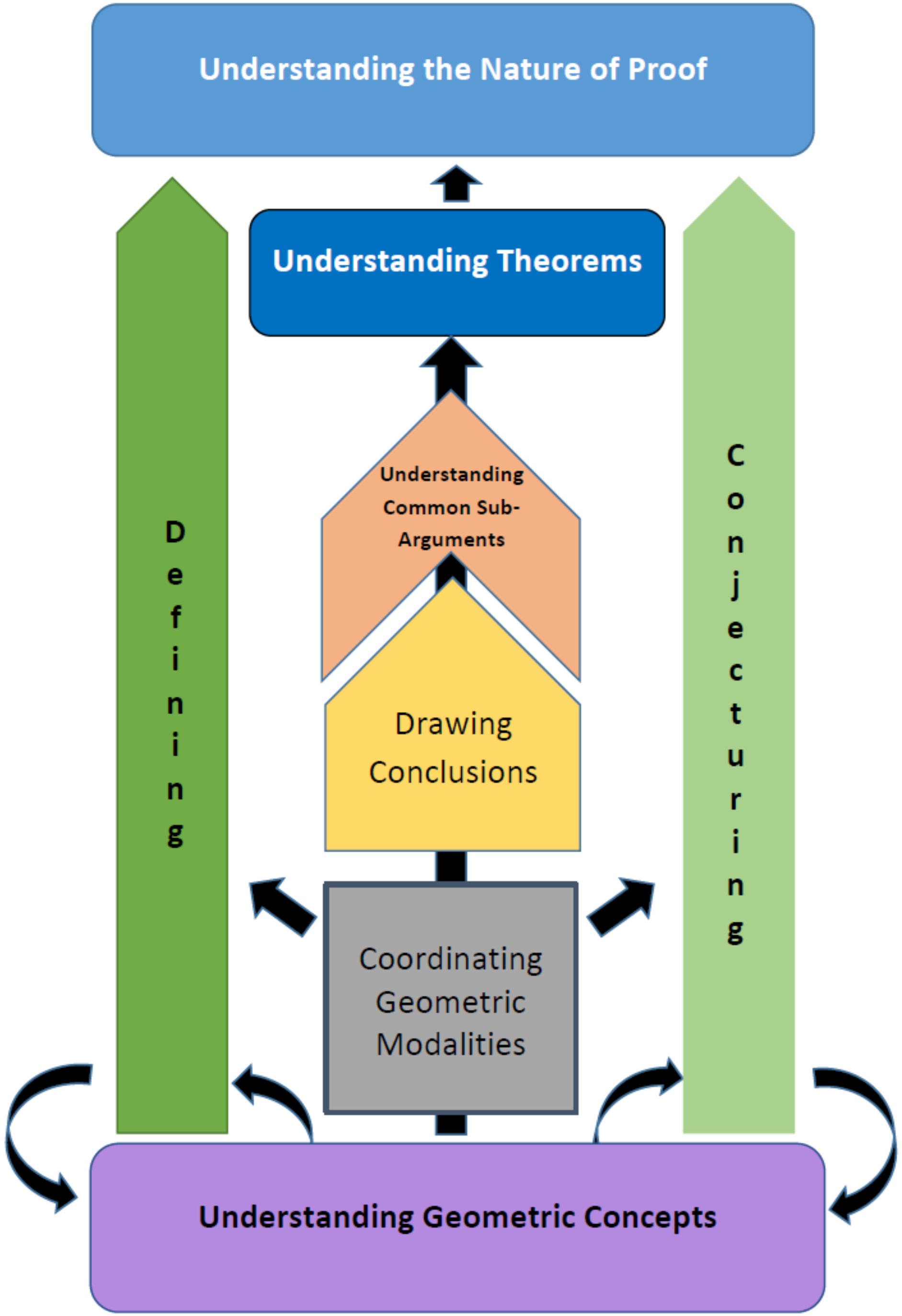
The Geometry Proof Scaffold





The Geometry Proof Scaffold

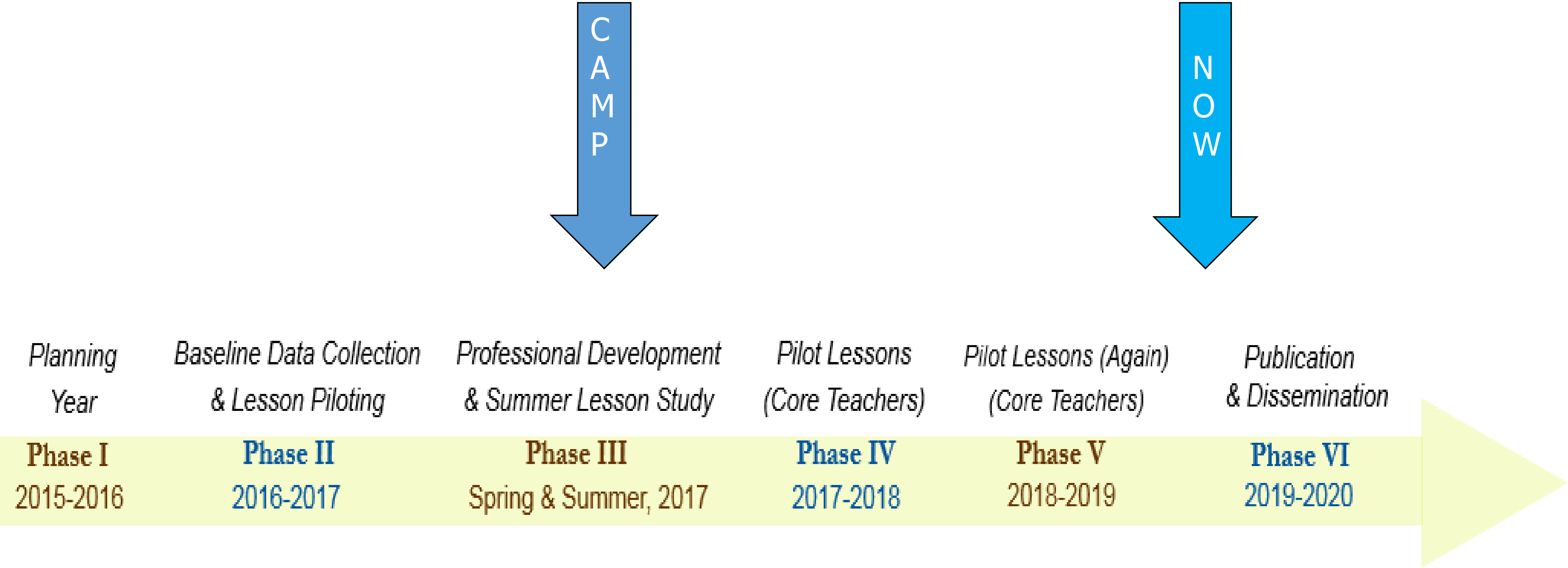
(i.e., the "GPS")



Geometry Proof Scaffold: A Pedagogical Framework for Teaching Proof

Sub-Goals	Descriptions	Competencies
Understanding Geometric Concepts	This sub-goal highlights the importance of understanding the building blocks of geometry.	1) Having accurate “mental pictures” of geometric concepts (i.e., having a concept image)
		2) Being able to verbally describe geometric concepts, ideally being fluent with one or more definitions of the concept (i.e., having or developing a concept definition)
		3) Determining examples and non-examples
		4) Understanding connections between classes of geometric objects, where they overlap, and how they are contained within other classes (i.e., understanding mathematical hierarchy)
Coordinating Geometric Modalities	This sub-goal highlights the ways in which the mathematics register draws on a range of modalities.	1) Translating between language and diagram
		2) Translating between diagram and symbolic notation
		3) Translating between language and symbolic notation
Defining	This sub-goal highlights the nature of definitions, their logical structure, how they are written, and how they are used.	1) Writing a “good” definition (includes necessary and sufficient properties)
		2) Knowing definitions are not unique (i.e., geometric objects can have different definitions)
		3) Understanding how to write and use definitions as biconditionals
Conjecturing	This sub-goal recognizes that conjecturing is an important part of mathematics and proving.	1) Understanding that empirical reasoning can be used to develop a conjecture but that it is not sufficient proof of the conjecture
		2) Being able to turn a conjecture into a testable conditional statement.
		3) Seeking out counterexamples to test conjectures and knowing that only one counterexample is needed to disprove a conjecture
		4) Understanding that when testing a conjecture, you are testing it for every case so you might begin by writing: “All,” “Every, or “For any”
Drawing Conclusions	This sub-goal presents the idea of an open-ended task that leads to conclusions that can be drawn from given statements and/or a diagram.	1) Understanding what can and cannot be assumed from a diagram
		2) Knowing when and how definitions and/or “Given” information can be used to draw a conclusion from a statement about a mathematical object
		3) Using postulates, definitions, and theorems (or combinations of these) to draw valid conclusions from some given information
Understanding Common Sub-arguments	This sub-goal recognizes that there are common short sequences of statements and reasons that are used frequently in proofs and that these pieces may appear relatively unchanged from one proof to the next.	1) Recognizing a sub-argument as a branch of proof and how it fits into the larger proof
		2) Understanding what valid conclusions can be drawn from a given statement and how those make a sub-argument (i.e., knowing some commonly occurring sub-arguments)
		3) Understanding how to write a sub-argument using notation and acceptable language (where “acceptable” is typically determined by the teacher)
Understanding Theorems	This sub-goal highlights the nature of theorems, their logical structure, how they are written, and how they are used.	1) Interpreting a theorem statement to determine the hypothesis and conclusion, and, if needed, providing an appropriate diagram
		2) If applicable, marking a diagram that satisfies the hypothesis of a proof
		3) Rewriting a theorem written in words in symbols and vice versa
		4) Understanding that a theorem is not a theorem until it has been proven
		5) Understanding that one cannot use the conclusions of the theorem itself to prove the conclusions of that theorem (i.e., avoiding circular reasoning)
		6) Understanding that theorems are mathematical statements that are only sometimes biconditionals
		7) Understanding the connection between logic and a theorem, for example, how to write the contrapositive of a conditional statement
Understanding the Nature of Proof	This sub-goal highlights the nature of proof, proof structure, and how the laws of logic are applied.	1) Understanding that the only way to sanction the truth of a conjecture is through deductive proof (rather than empirical reasoning)
		2) Exploring a pathway for constructing a proof (i.e., the problem solving aspect of proving)
		3) Understanding that proofs are constructed using axioms, postulates, definitions, and theorems and that they follow the laws of logic
		4) Knowing what language is acceptable to use and how to write up a proof
		5) Recognizing that if you prove that something is true for one particular geometric object, then it is true for all of them

PISC Project Timeline



PISC Lesson Plans

PISC LESSONS 1-8	
1	Getting Started in Euclidean Geometry
2	Investigating Geometric Concepts
3	Developing Definitions
4	Coordinating Geometric Modalities – Day 1
5	Coordinating Geometric Modalities – Day 2
6	Coordinating Geometric Modalities – Day 3
7	Drawing Conclusions – Day 1
8	Drawing Conclusions – Day 2

PISC LESSONS 9-16	
9	Deductive Structure
10	Proving Simple Theorems
11	Common Sub-Arguments
12	Hidden Triangles – Day 1
13	Hidden Triangles – Day 2
14	First Triangle Proofs
15	Conjecturing about Parallelograms – Day 1
16	Conjecturing about Parallelograms – Day 2

Five Misconceptions/Errors Addressed in the PISC Materials

Data Sources

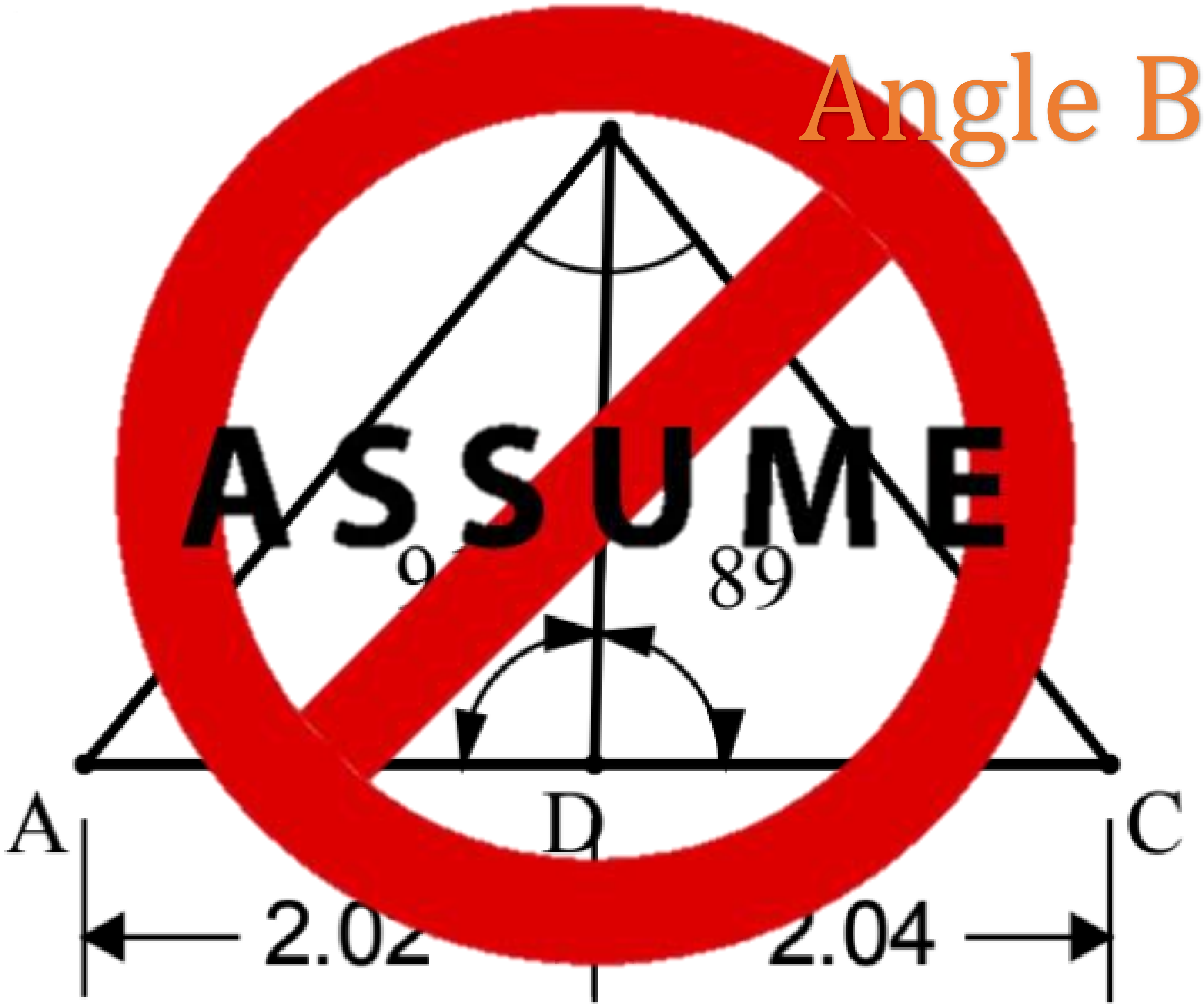
- Over 150 hours of classroom observations of teaching proof in geometry
- Over 40 interviews with teachers of proof in geometry
- Clinical interviews with 29 students who earned As and Bs in their geometry proof units
- End-of-course post-test results from an 11-item assessment focused on proof in geometry (n = 389)
- Data (written work and videos) from a 2-week Summer Geometry Institute (SGI) with 11 students who were scheduled to study geometry proof in the upcoming year

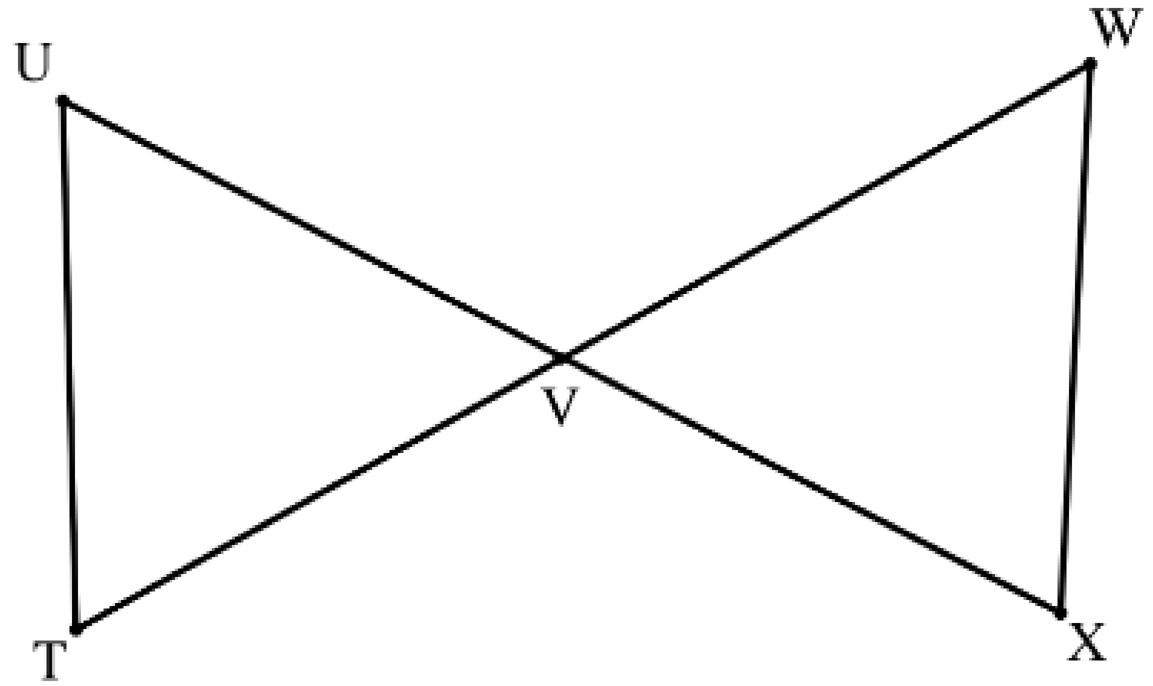
#1: You can draw conclusions from diagrams.

Given: $\overline{BD}$ bisects $\angle ABC$

Angle Bisector

$\cong \angle CBD$





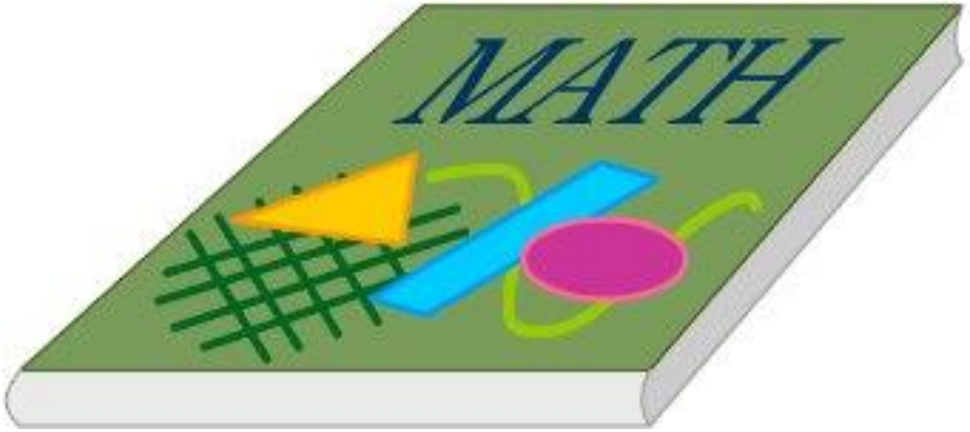
$\overline{UX}$ bisects $\overline{TW}$ at V

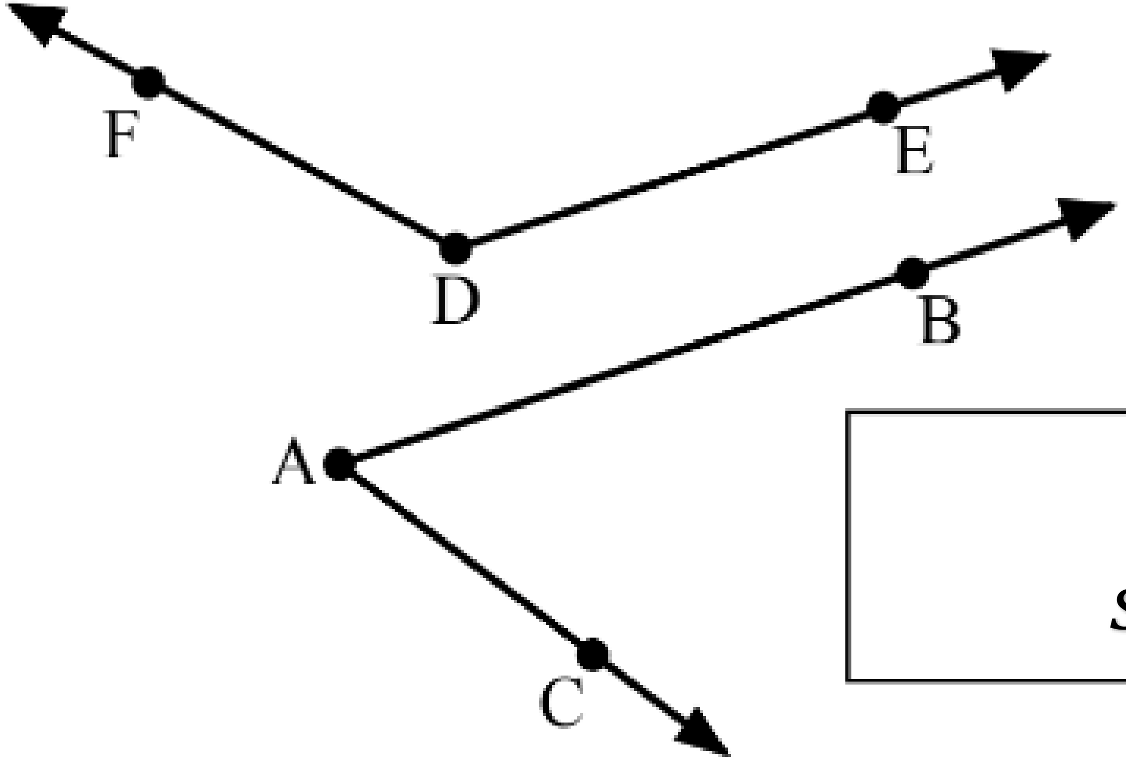
(Given)

↓

V is the midpoint of $\overline{TW}$

(Definition of line segment bisector)



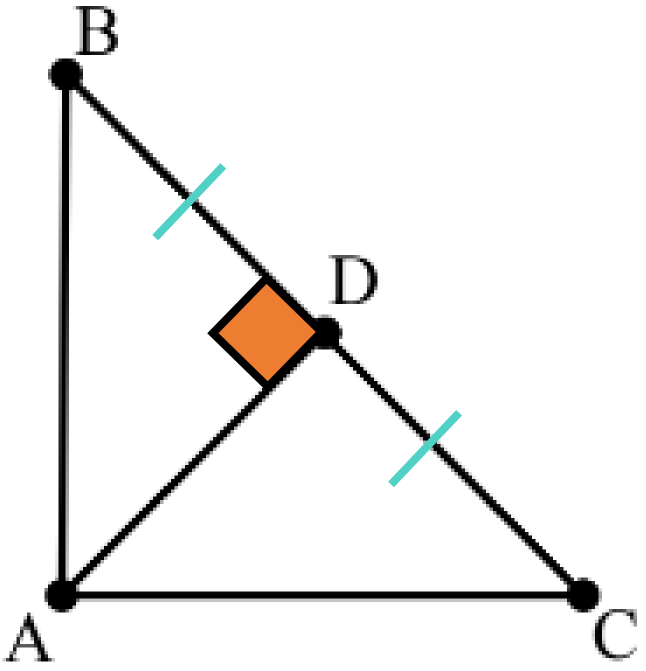


$\angle FDE$ *and* $\angle BAC$ *are supplementary angles*

(Given)

$$m\measuredangle FDE + m\measuredangle BAC = 180^\circ$$

(Definition of Supplementary Angles)



$\overline{AD}$ *is the* $\perp$ *bisector of* $\overline{BC}$

(Given)

$\angle ADC$ *and* $\angle ADB$ *are right angles*

(Definition of Perpendicular Lines)

Right Angles Are Congruent (THEOREM)

D is the midpoint of $\overline{BC}$

(Definition of Line Segment Bisector)

$$\overline{BD} \cong \overline{CD}$$

(Definition of Midpoint)

Addressing #1: You can draw conclusions from diagrams.

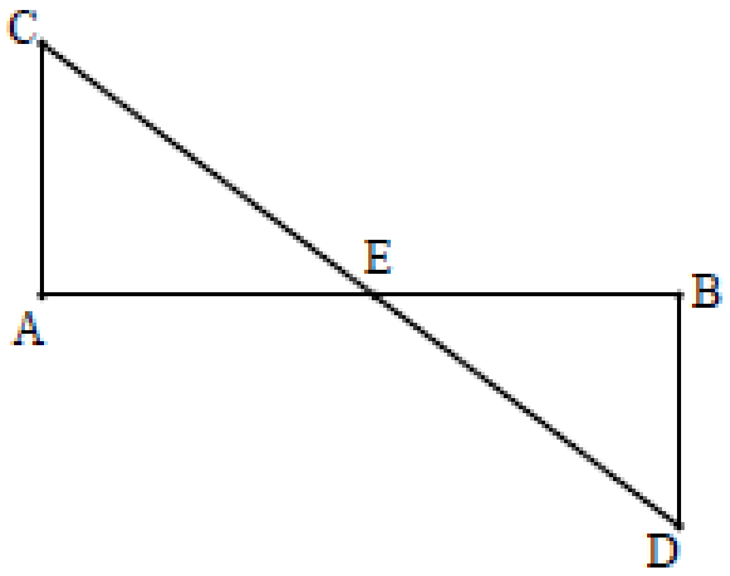
Teach students to draw valid conclusions *before* teaching proof.

2: You cannot make assumptions about diagrams.

Assumptions about Diagrams

You **may** assume…		You **may not** assume…	
Straight Lines and Angles	If lines and angles look straight, they are.	*Congruent angles and segments*	If angles or line segments look congruent, they may not be.
Collinearity	If points look collinear, they are.	*Perpendicular lines*	If lines look perpendicular, they may not be.
Relative Location of Points	If a point looks like it is to the left or right of another point, it is.	*Parallel Lines*	If lines look parallel, they may not be.
Betweenness of Points	If a point looks like it is between two other points, it is.	*Relative Size of Angles and Segments*	If an angle or line segment looks bigger or smaller than another, it may not be.
Intersection of Lines	If lines look like they intersect at a point, they do.		
Adjacent Angles	If angles look adjacent, they are.	*Right Angles*	If an angle looks like a right angle, it may not be.

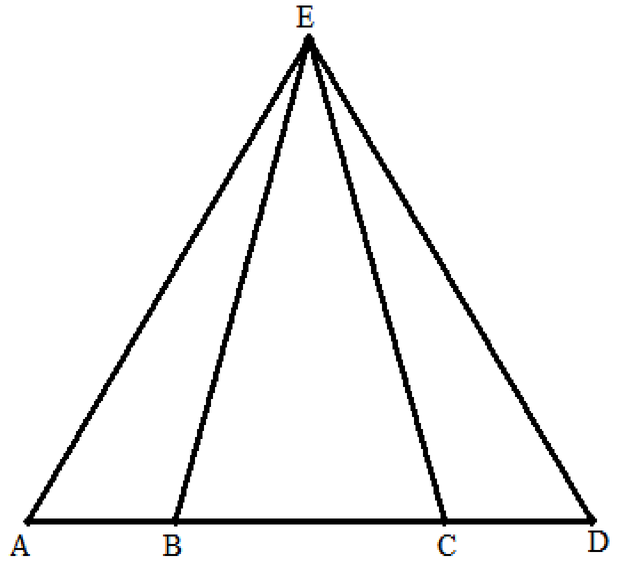
Given the diagram below, list some things that you may assume and what you may not assume.



You may assume	You may not assume

Summer Geometry Institute (i.e., Geometry Camp) Formative Assessment

State two things that you can conclude from the diagram and two things that you cannot.

	Things I can assume	Things I cannot assume
	Points BC are on $\overline{AD}$	$\overline{AE} = \overline{BE}$
	that $\overline{AE}$ $\overline{DE}$ $\overline{BE}$ $\overline{CE}$ connect	$\overline{CE} = \overline{DE}$

Addressing # 2: You cannot make assumptions about diagrams.

↓

Teach students explicitly what they can and cannot conclude about diagrams.

#3: A definition can include all of the properties that one knows about the geometric object.

Students practice writing definitions

Questions to Consider When Defining

1. What is the geometric object (e.g., a point, a line segment, a triangle, a quadrilateral)? Consider all options.
2. What is special about this particular object (i.e., what makes it different from other similar objects)?
3. Did you consider possible counterexamples that would indicate that your definition is inaccurate?
4. Is the definition economical[3] (i.e., did you include all of the important information, but not too much)?
5. Does the definition make sense as a biconditional statement?

Task: Define an isosceles triangle

Jackson: An isosceles triangle is a triangle with two line segments that have equal length.

Shar: Shouldn't you say sides?

Isosceles Triangle

1. An isosceles triangle is a(n) triangle

(What is this geometric object?)

that ~~has two sides w/ equal lengths~~.

(What is special about this particular object?)

Task: Define an isosceles triangle

Jackson: An isosceles triangle is a triangle with two line segments that have equal length.

Shar: Shouldn't you say sides?

Isosceles Triangle

1. **An isosceles triangle** is a(n) triangle
(What is this geometric object?)

that has 2 congruent sides.
(What is special about this particular object?)

Summer Camp

2. Use what you learned about writing good definitions in mathematics to complete the tasks below.

Define a pentagon.

A geometric shape is a pentagon if and only if it has 5 sides.

Jenna's teacher asked her to define a rectangle. What do you think of Jenna's definition below? Would this qualify as a "good" definition? Why or why not?

A rectangle is a four-sided figure that is also a parallelogram with four straight sides, and it has two short sides, two long sides, four right angles and the opposite sides are parallel.

I would say she doesn't have a good definition since her definiton wasn't economical she should change four sided figure to quadrilateral and could get rid of four straight sides. and she should reword the last order [illegible]

Addressing #3: A definition can include all of the properties that one knows about the geometric object.

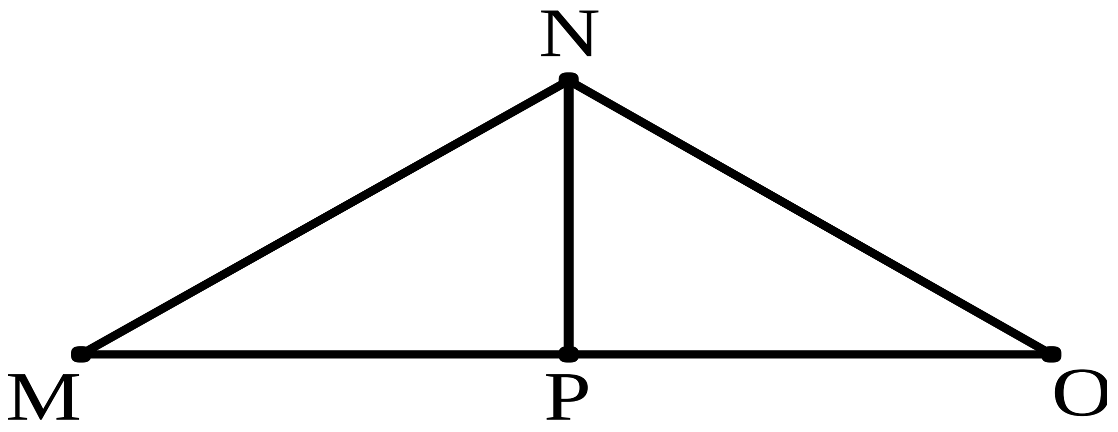
↓

Have students practice defining and continually emphasize the importance of knowing definitions.

#4: Bisectors divide triangles in half or act as lines of symmetry.

Students work on Understanding Geometric Concepts and Drawing Conclusions around these ideas…

Given: $\overline{NP}$ bisects ∠MNO

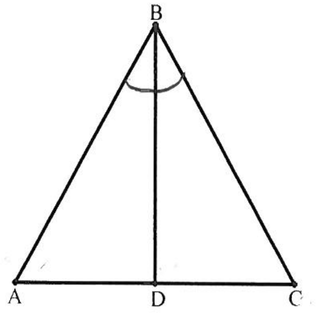


What can you conclude from this Given information?

True

1. Use the figure below to determine what you could conclude if you were "Given" each of the statements in the various situations.

Figure:



Situation 1:

$\overline{BD}$ bisects $\angle ABC$

(Given)

↓

$\angle ABD \cong \angle CBD$

(Definition of Angle Bisector)

Situation 2:

$\overline{BD}$ bisects $\overline{AC}$

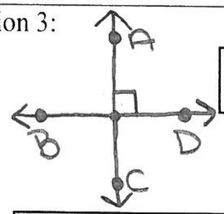
(Given)

↓

D is the midpoint of $\overline{AC}$

(Definition of Line Segment Bisector)

Situation 3:



$\overline{BD}$ is the perpendicular bisector of $\overline{AC}$

(Given)

↓

$\overline{BD} \perp \overline{AC}$

(Definition of Perpendicular Bisector)

Addressing #4: Bisectors divide triangles in half or act as lines of symmetry.

↓

Focus on the three types of bisectors repeatedly, and formatively assess students' progress.

#5: When attempting to prove a conjecture as a theorem, one assumes the conclusion of the statement.

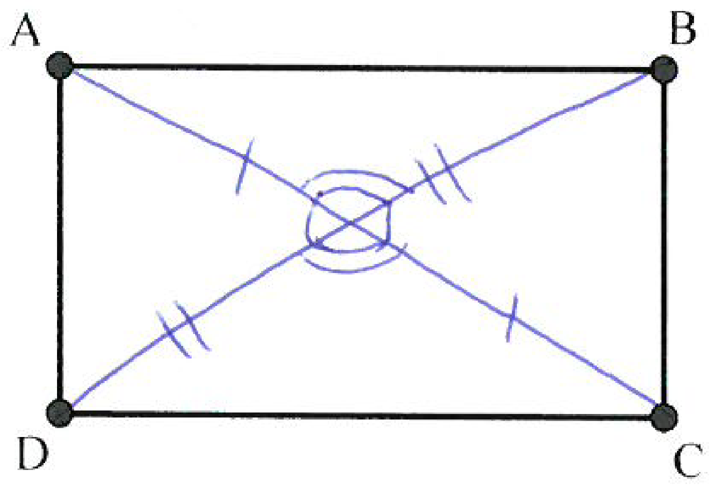
Suppose you developed the following conjecture and drew the diagram below:

Conjecture: *The diagonals of a rectangle are congruent.*

Write the "Given" and the "Prove" statements that you would need to prove your conjecture.

Given: The diagonals of a rec. are congruent

Prove: ______



Initially allow students to write their conjectures using their own language…

- Rewrite the conjecture, "**The diagonals of a parallelogram bisect each other**," as an "If…, then…" statement.

Addressing #5: When attempting to prove a conjecture as a theorem, one assumes the conclusion of the statement.

Teach students to rewrite conjectures as conditional statements and identify the hypothesis as the "Given" and the conclusion as the "Prove" statement.

Conclusions

- Students’ difficulties in learning proof may stem from inadequate exposure to geometry.
- We can provide level-appropriate scaffolding to help our students avoid an abrupt transition to deductive proof.
- We can break down the skills needed for proofs, and in so doing, flesh out the “shallow end.”

Conclusions

- Addressing students’ misconceptions through strategic tasks can support students in being better prepared to write proof in geometry.

- When common misconceptions are addressed, students are less resistant to learning proof and teachers have greater success in teaching proof.

Conclusion

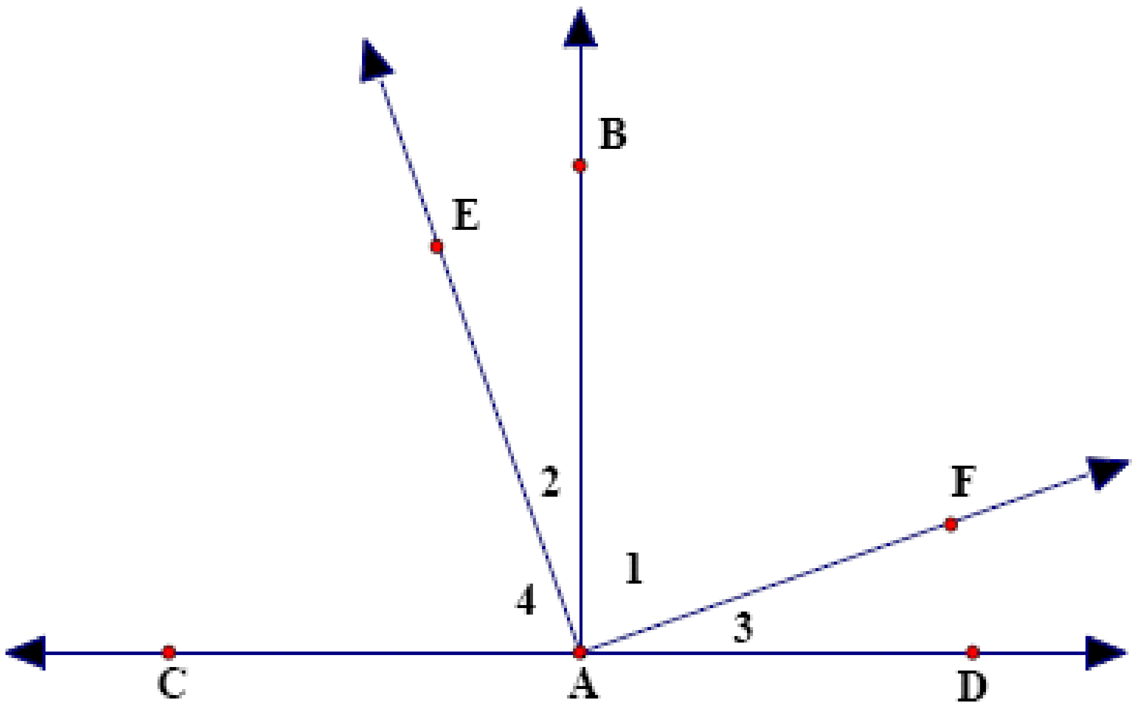
PROOF CAN BE TAUGHT!

Thank you!

This material is based upon work supported by the National Science Foundation (NSF; Award #1453493, PI: Michelle Cirillo). Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the NSF. For more information about the project visit: www.pisc.udel.edu.

Email mcirillo@udel.edu for questions about or updates on the project.
Follow me on Twitter: @UDMichy

Complete the proof below.



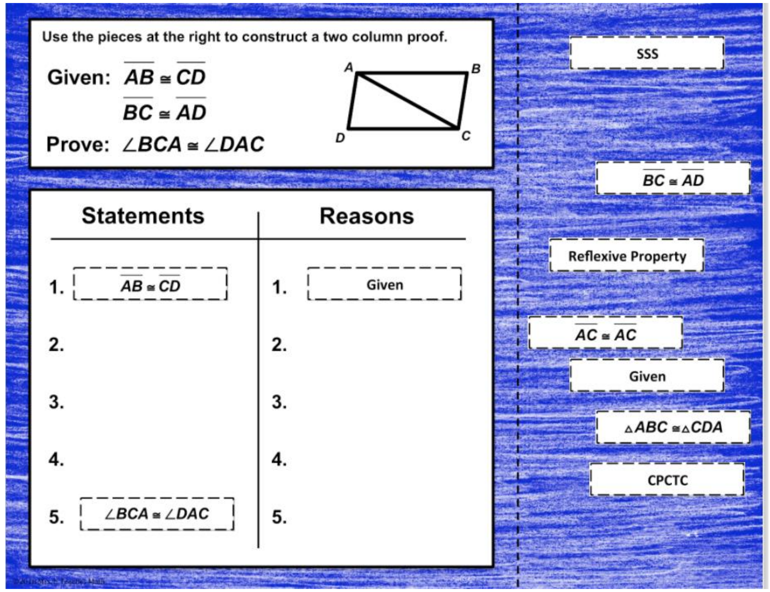
Given:

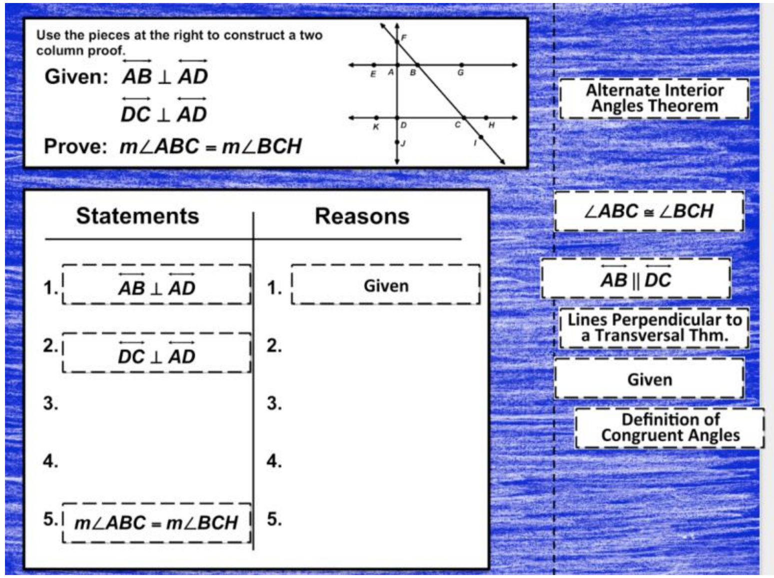
- $\overrightarrow{AB} \perp \overrightarrow{AC}$ and $\overrightarrow{AB} \perp \overrightarrow{AD}$
- $\overrightarrow{AE} \perp \overrightarrow{AF}$
- $\overrightarrow{AE}$ lies in the interior of $\angle CAB$
- $\overrightarrow{AF}$ lies in the interior of $\angle BAD$

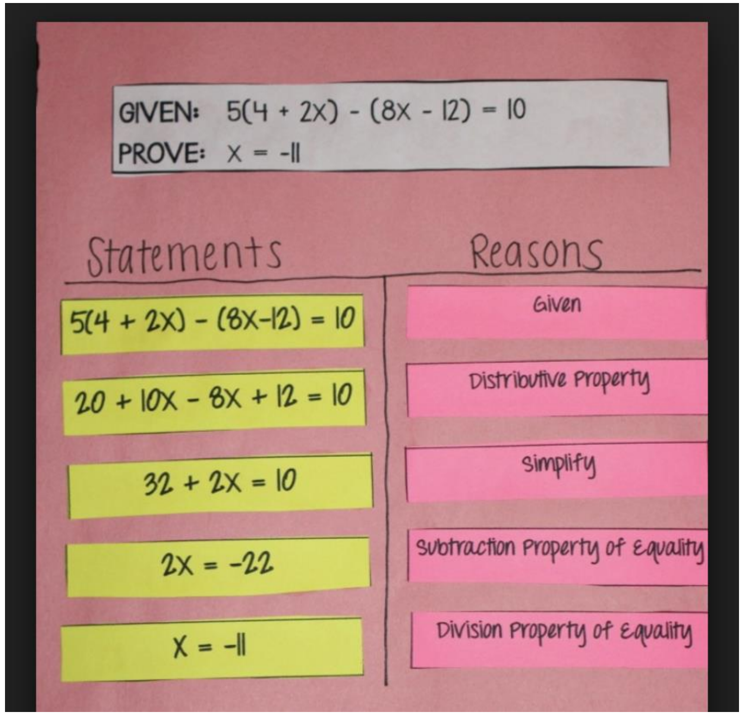
Prove:

$\angle 3$ and $\angle 4$ are complementary

Statements	**Reasons**
1. $\overrightarrow{AB} \perp \overrightarrow{AC}$ and $\overrightarrow{AB} \perp \overrightarrow{AD}$	1. Given


Use the pieces at the right to construct a two column proof.
Given: $\overline{AB} \cong \overline{CD}$
$\overline{BC} \cong \overline{AD}$
Prove: $\angle BCA \cong \angle DAC$
A
B
D
C
Statements
Reasons
1. $\overline{AB} \cong \overline{CD}$
1. Given
2.
2.
3.
3.
4.
4.
5. $\angle BCA \cong \angle DAC$
5.
SSS
$\overline{BC} \cong \overline{AD}$
Reflexive Property
$\overline{AC} \cong \overline{AC}$
Given
$\triangle ABC \cong \triangle CDA$
CPCTC


Use the pieces at the right to construct a two column proof.
Given: $\overleftrightarrow{AB} \perp \overleftrightarrow{AD}$
$\overleftrightarrow{DC} \perp \overleftrightarrow{AD}$
Prove: $m\angle ABC = m\angle BCH$
Statements
Reasons
1. $\overleftrightarrow{AB} \perp \overleftrightarrow{AD}$
1. Given
2. $\overleftrightarrow{DC} \perp \overleftrightarrow{AD}$
2.
3.
3.
4.
4.
5. $m\angle ABC = m\angle BCH$
5.
Alternate Interior Angles Theorem
$\angle ABC \cong \angle BCH$
$\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$
Lines Perpendicular to a Transversal Thm.
Given
Definition of Congruent Angles


GIVEN: 5(4 + 2X) - (8X - 12) = 10
PROVE: X = -11
Statements
Reasons
5(4 + 2X) - (8X-12) = 10
Given
20 + 10X - 8X + 12 = 10
Distributive Property
32 + 2X = 10
Simplify
2X = -22
Subtraction Property of Equality
X = -11
Division Property of Equality

Common Sub-Arguments

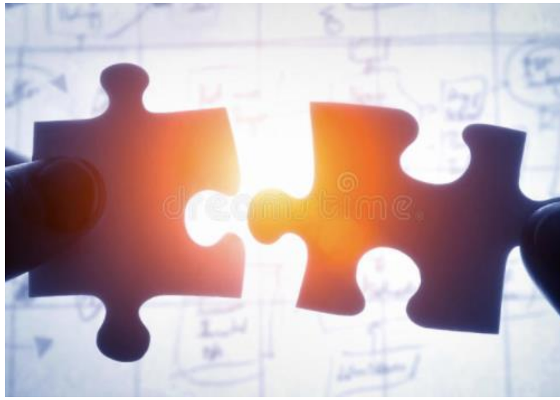
Line Segment Bisector

Perpendicular Lines

Vertical Angles

Alternate Interior Angles

Linear Pair



Five Common Sub-Arguments

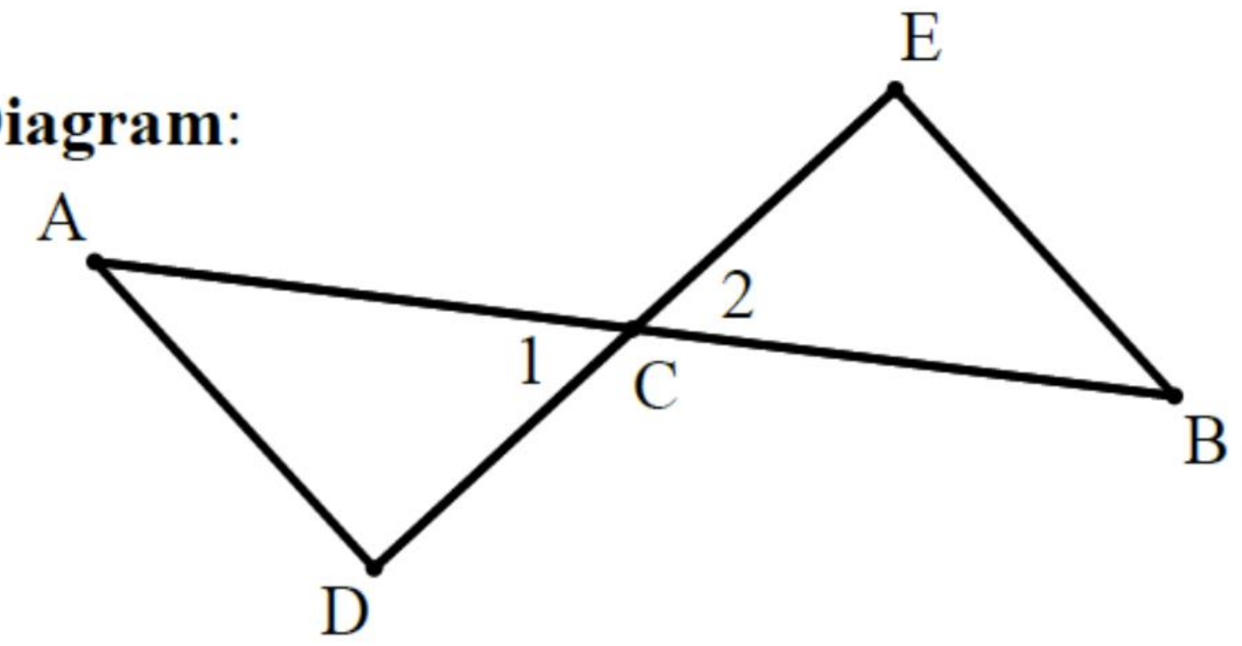
Student Task

Given: $\overline{AB}$ bisects $\overline{DE}$

$\overline{AD} \perp \overline{DE}$

$\overline{BE} \perp \overline{DE}$

Diagram:



Prove: $\triangle ADC \cong \triangle BEC$

Student Task

Sub-Arguments

Line Segment Bisector

Given: $\overline{AB}$ bisects $\overline{DE}$

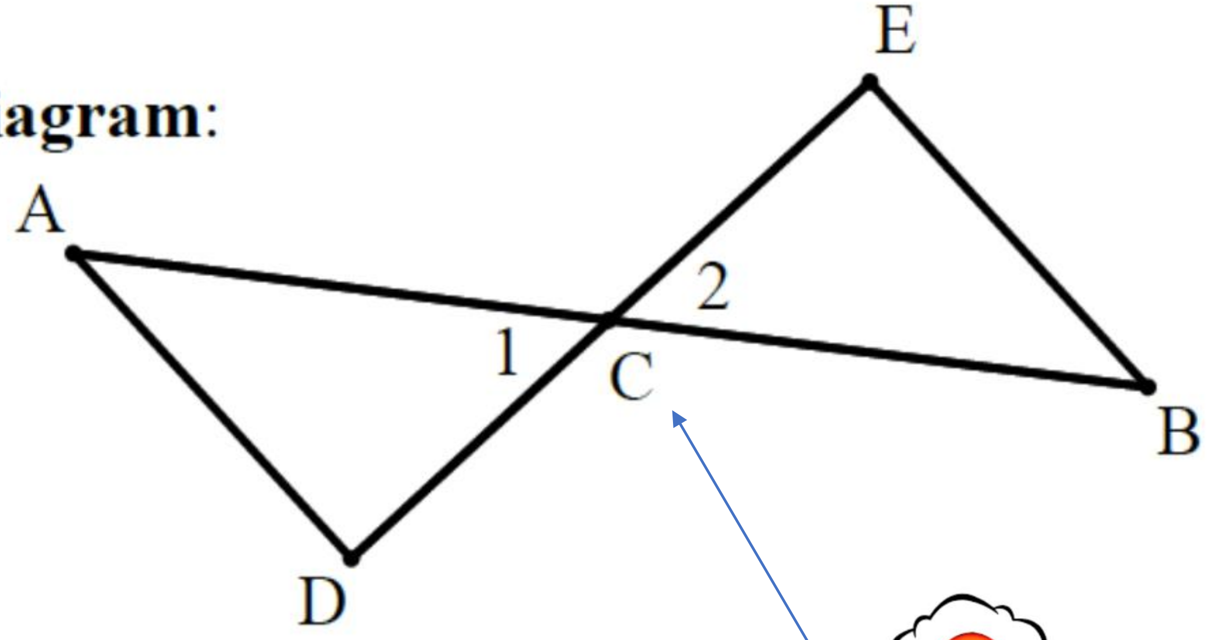
$\overline{AD} \perp \overline{DE}$

$\overline{BE} \perp \overline{DE}$

Perpendicular Lines

e: $\triangle ADC \cong \triangle BEC$

Diagram:

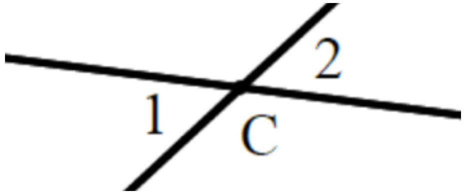


Vertical Angles

$\overline{AB}$ bisects $\overline{DE}$

Sub-Argument 1

Line Segment Bisector



Sub-Argument 3

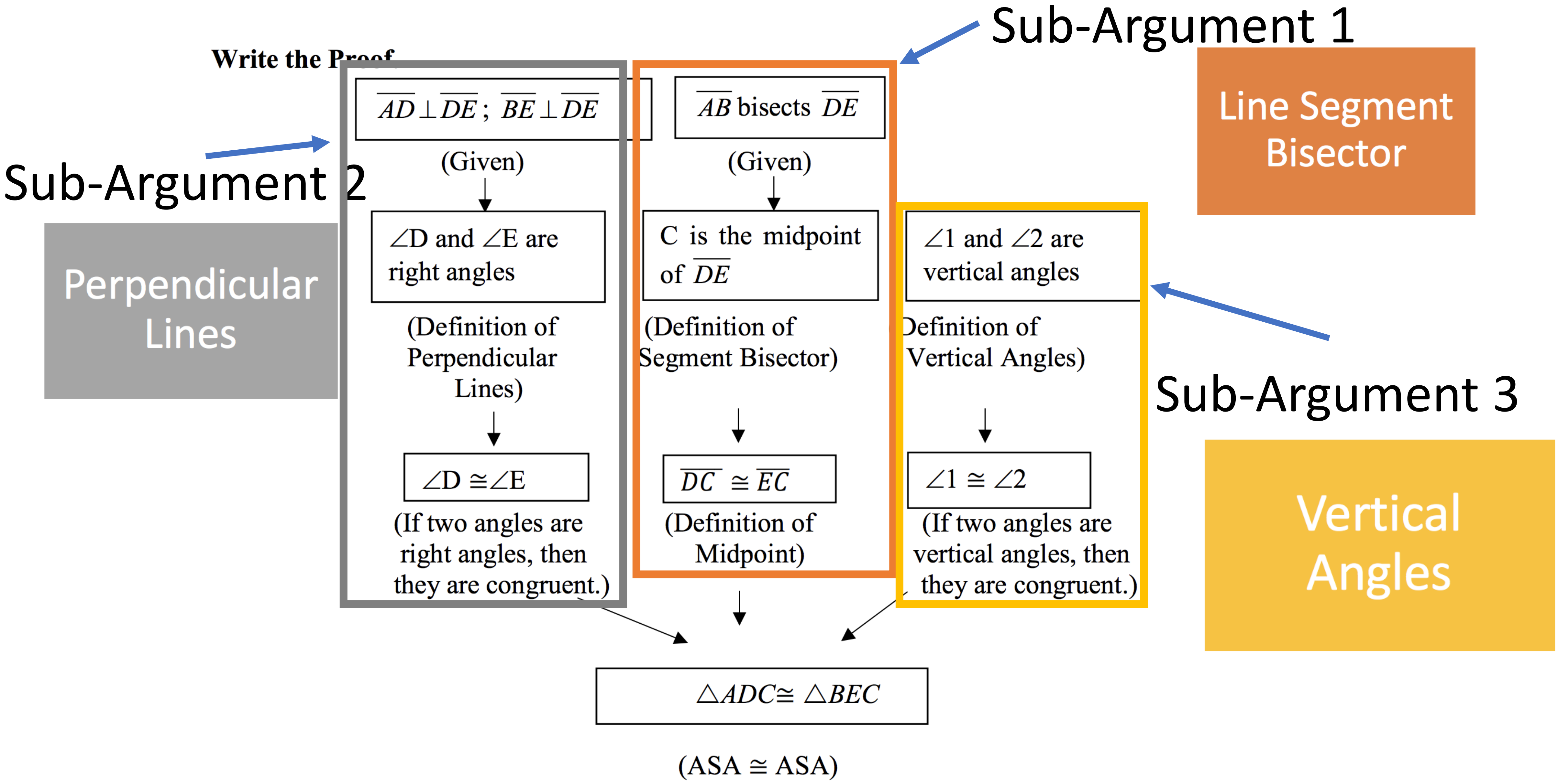
Vertical Angles

$\overline{AD} \perp \overline{DE}$

$\overline{BE} \perp \overline{DE}$

Sub-Argument 2

Perpendicular Lines


Write the Proof
Sub-Argument 1
Line Segment Bisector
Sub-Argument 2
Perpendicular Lines
Sub-Argument 3
Vertical Angles
$\overline{AD} \perp \overline{DE}$; $\overline{BE} \perp \overline{DE}$
(Given)
∠D and ∠E are right angles
(Definition of Perpendicular Lines)
∠D ≅ ∠E
(If two angles are right angles, then they are congruent.)
$\overline{AB}$ bisects $\overline{DE}$
(Given)
C is the midpoint of $\overline{DE}$
(Definition of Segment Bisector)
$\overline{DC} \cong \overline{EC}$
(Definition of Midpoint)
∠1 and ∠2 are vertical angles
Definition of Vertical Angles)
∠1 ≅ ∠2
(If two angles are vertical angles, then they are congruent.)
$\triangle ADC \cong \triangle BEC$
(ASA ≅ ASA)

Sub-Argument 2

Perpendicular Lines

Statements	Reasons
1. $\overline{AD} \perp \overline{DE}$; $\overline{BE} \perp \overline{DE}$	1. Given
2. $\angle$D and $\angle$E are right angles	2. Definition of Perpendicular Lines
3. $\angle D \cong \angle E$	3. If two angles are right angles, then they are congruent.
4. $\overline{AB}$ bisects $\overline{DE}$	4. Given
5. C is the midpoint of $\overline{DE}$	5. Definition of Line Segment Bisector
6. $\overline{DC} \cong \overline{EC}$	6. Definition of Midpoint
7. $\angle 1$ and $\angle 2$ are vertical angles	7. Definition of Vertical Angles
8. $\angle 1 \cong \angle 2$	8. If two angles are vertical angles, then they are congruent.
9. $\triangle ADC \cong \triangle BEC$	9. ASA $\cong$ ASA

Sub-Argument 1

Line Segment Bisector

Sub-Argument 3

Vertical Angles