CHAPTER 2

Exploring the Viral Spread of Disease and Disinformation

Samuel Otten
University of Missouri, Columbia MO

Julia Bemke
Passaic County Technical Vocational Schools, Wayne NJ

Jerred Webb
Waynesville R-VI School District, Waynesville MO

The worldwide COVID-19 pandemic has highlighted the importance of mathematical models in predicting the spread of the coronavirus (Srinivas 2020; Stevens & Muyskens 2020) and assessing the effectiveness of various safety measures in reducing that spread (Li et al 2020). These models can be extremely sophisticated, drawing on the expertise of applied mathematicians, epidemiologists, public health experts, and others, but at its core, there is a notion of exponential growth that is relevant for the secondary mathematics curriculum. It is crucial that students recognize how an exponential situation is a different sort of threat than a linear or polynomial situation, and the pandemic provides heartbreaking motivation for this point.

Yet the pandemic has also revealed that we are not only threatened by an airborne virus, we are also threatened by the spread of misinformation (false or misleading information that is shared by someone who believes it to be accurate) and disinformation (false or misleading information that is shared knowingly and purposefully). For example, in July 2020, before vaccines were available and during a time when mask-wearing was highly recommended by experts, a video went viral falsely claiming that COVID-19 could be cured and that people did not need to wear masks (Shead, 2020). It was viewed tens of millions of times before the social media companies pulled it down. With the emergence of the vaccines, there have been many more misleading stories that have gone viral. Indeed, throughout the pandemic, many of the most viral stories on social media have contained misinformation (Parks, 2021). In some ways, this viral spread of misinformation has a similar mathematical structure to the spread of an airborne virus and it is important that our students, to the extent possible, are inoculated against misinformation and unfounded conspiracy theories.

The tasks described in this chapter are intended to build connections between these real-world dangers of viral spread and some relevant topics from the secondary mathematics curriculum. We also explore a link between mathematical reasoning and media literacy—the ability to discern the commercial, ideological, or political motivations of media and the recognition that receivers negotiate the meaning of messages (Aufderheide, 1993)—so that, just as we know to take safety precautions with regard to an airborne coronavirus, we can also help our students learn to take precautions against the spread of misinformation on social media.

Overview of the Tasks

We are certainly not the first people to note the connection between viral spread and exponential growth. Other educators have already developed lessons that deal with the exponential nature of the coronavirus pandemic (e.g., Chiucarello 2020; Young-Saver 2021) and the use of proportional reasoning to compare the severity of COVID-19 outbreaks in different regions (e.g., www.nctm.org/Coronavirus-and-Pandemics-Math-Resources/). In our first task, we intend to complement these existing lessons by not only using mathematical tools to track and predict the exponential spread, but also place the students into the scenarios so that they can think about how our collective behaviors “bend the curve” (or not). We then engage students in a detailed comparison and contrast of the models that result from different choices of behavior and this allows them to make connections to function operations such as the subtracting of one function from another to estimate the number of hospitalizations avoided by a particular safety choice.

In our second task, we shift from an exponential, holistic perspective on COVID-19 to a more personal perspective thinking about risks and probabilities. We situate this task in a fictionalized reality that is inspired by real issues related to estimating risk. Throughout the pandemic, people have had inaccurate estimations of various risk probabilities (Kramer 2021), both underestimating some things that are risky ("eating in a restaurant is about as risky as going to grocery store" when actually a restaurant is much more dangerous) and overestimating some things that are fairly safe ("wearing a mask will harm me because of increased carbon dioxide levels" when actually there is no real risk from the mask). People also sometimes misconstrue the probabilities related to safety precautions. We provide a fictionalized context where students can carefully think through how various safety precautions relate to one another in terms of risk probability. As with the first task, we want students to feel like active decision makers in navigating health risks and to be thoughtful about the outcomes of those decisions, with mathematics as a helpful tool in their thinking. In this way, we are striving for mathematical literacy as it relates to realistic (if fictional) scenarios, rather than mere mathematical skills and knowledge with regard to exponents and probability. We also seek to strengthen students' health literacy (the capacity to obtain, process, and understand the information needed to make appropriate health decisions; www.hrsa.gov/about/organization/bureaus/ohe/health-literacy/index.html).

In our third task, we seek to connect mathematics with media literacy. We focus on some strategies that can help assure that each of us is participating in the spread of information and not the spread of misinformation. Literacy tools have been found to stem the tide of misinformation, such as asking "how do you know that?" and "is there any way this could be wrong?”. These are very fitting for a mathematics classroom. This task invites students to use these questions to carefully think through claims about mathematical truths, and then there are opportunities to extend this so that we all strive to be more diligent in discerning information from misinformation in everyday life.

These three tasks are designed to be completely separable. We think that they contain worthwhile mathematics and can be implemented on their own, even though there are connections around the idea of exponential spread and also around the notion of using mathematical tools to understand the repercussions of our choices. Because of these connections, the three tasks could be used in conjunction to emphasize and develop mathematical literacy. Here, students employ mathematics to solve problems in real-world contexts, explain and predict real-world phenomena, and discuss how mathematical reasoning can play a role in the world, helping us to “make the well-founded judgments and decisions needed by constructive, engaged and reflective 21st century citizens” (PISA and this volume).

Classroom Context

We wrote these tasks imagining a classroom that emphasizes the standards for mathematical practice (Koestler et al. 2013). Student ideas and sense-making should be welcomed and teachers should consistently be inviting students to persevere in problem solving, construct arguments, attend to the precision of their communications, and look for structures. These particular tasks also entail reasoning that moves between the problem context and the mathematical ideas at play, so both arenas of reasoning should be valued. There are also opportunities for strategic uses of technology, such as spreadsheets for data management and calculations as well as graphing calculators to aid students in thinking about functions and curves.

Because these tasks are directly or indirectly related to the COVID-19 pandemic, it is important to be mindful of the personal impact of the disease on your own students. It is extremely likely that they know of someone who has had COVID-19 but there is also a substantial likelihood that someone in the class has had a family member or friend who has been hospitalized or possibly died from the disease. Furthermore, there is a possibility that some students or their family members may be caught up in misinformation about the pandemic and so they may react negatively to data about the protections afforded by masks or vaccinations. For all these reasons, we must approach the context with care and compassion and you may even need to modify or avoid it altogether if it is too emotionally fraught. Students' connections with the pandemic can make it relevant for their learning, but it can also conjure feelings that will prevent meaningful learning. This is yet another delicate balance that teachers must maintain as they continue the heroic work of pandemic teaching.

Task 1: Changing the COVID Case Curve

COVID-19 is a disease caused by a new strain of the coronavirus. This disease reached the United States in early 2020 and deaths from COVID-19 started being systematically reported in March 2020. This table contains the number of COVID-19 cases and COVID-19 deaths in the USA for one month near the start of the pandemic.

Date	COVID-19 New Cases	COVID-19 New Deaths
3/2/2020	16	3
3/3/2020	21	4
3/4/2020	36	2
3/5/2020	67	0
3/6/2020	83	3
3/7/2020	117	4
3/8/2020	119	3
3/9/2020	201	4
3/10/2020	270	5
3/11/2020	245	6

3/12/2020	405	6
3/13/2020	556	7
3/14/2020	674	10
3/15/2020	702	8
3/16/2020	907	23
3/17/2020	1399	26
3/18/2020	2444	45
3/19/2020	4043	50
3/20/2020	5619	65
3/21/2020	6516	83
3/22/2020	8545	98
3/23/2020	10432	121
3/24/2020	10433	206
3/25/2020	14634	269
3/26/2020	16998	299
3/27/2020	17330	417
3/28/2020	18100	530
3/29/2020	18520	418
3/30/2020	21469	650
3/31/2020	24506	936
4/1/2020	26930	1021

https://www.nytimes.com/interactive/2021/us/covid-cases.html

Part One: Considering Different Safety Measures

1. Use this exponential function that represents COVID-19 cases in the USA for the first month of the pandemic:

$$cases = 1.39^{days} + 14$$

where *days* means days since February 28, 2020 (so *days* = 2 means March 2, 2020).

   a. What does the constant term 14 represent in the context of COVID-19 spread?

   b. What does the 1.39 exponential base represent?

   c. Would you expect this model to be very accurate for the weeks and months beyond April 2020 (beyond the given data set)?

2. One of the precautions considered at the start of the COVID-19 pandemic was to ban travel from certain countries like China, but COVID-19 was already in the United States at the time. Let's suppose that a foreign travel ban had reduced the number of

Americans returning from abroad or the number of foreign citizens entering the United States and so there were fewer initial cases of COVID-19. What if there were only 7 initial cases instead of 14? How would this change the spread of COVID-19 during the first month?

3. Masks, if worn properly, can prevent up to 70% of COVID-19 cases (Howard et al. 2021). But across a wide population it is hard to have everyone wear masks properly so 70% prevention is not very realistic. Nevertheless, general mask wearing has been estimated to reduce approximately 10% of the spread of COVID-19 (Abaluck et al. 2021). What if the infectious rate in the function had been reduced by 10%? How would this change the spread of COVID-19 during the first month?

Part Two: Comparing Two COVID-19 Scenarios

4. For this exploration, you can again use the exponential function:

$$cases = 1.39^{days} + 14$$

where *days* means days since February 28, 2020. Your task is to also make a new exponential function to compare with the original one. Propose your own set of safety measures and estimate how they may reduce the spread of COVID-19. These could be government mandates but they do not have to be. Your safety measures could also be advertising campaigns or social media influence (e.g., encouraging masks, encouraging outdoor vacations instead of indoor gatherings), private business decisions (e.g., restaurants closing their dining room and only doing takeout), or other initiatives. Based on your proposed safety measures, create a new exponential function that represents the spread of COVID-19 for the first month of the pandemic if your ideas were in effect.

5. Compare your new exponential function to the original exponential function.
    a. How different are the two functions at the start of March 2020?

b. How different are the two functions by April 2020?

c. Can you think of a function that takes as input the number of *days* since February 28, 2020, and gives as output the number of people saved from getting COVID-19 by your safety precautions? If we assume that 1.5% of people who get COVID-19 die while infected with the disease, how many lives would be saved by your hypothetical safety precautions?

Standards

- CCSSM Practice Standards
  - MP2 Reason abstractly and quantitatively.
  - MP4 Model with mathematics.
  - MP5 Attend to precision.
- CCSSM Content
  - **HSF-LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.
  - **HSF-IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
  - **HSF-IF.8b** Use the properties of exponents to interpret expressions for exponential functions.
  - **HSF-IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
  - **HSF-BF.1** Write a function that describes a relationship between two quantities and **HSF-BF.1b** Combine standard function types using arithmetic operations.

Situating the Task

This task primarily focuses on function standards at the high school level. It is best suited for an Algebra 2 course or its equivalent, but modifications could be made to adapt it for an Algebra 1 setting or for a Precalculus course. We presume conceptual understanding of functions and we envision the task as an opportunity to connect components of an exponential function to a realistic situation. Students explore how variations in those components affect the function and think about functions not just as a formula for inputs and outputs but as a mathematical object that can be revised, critiqued, and compared to or combined with other functions. We presented the task in two parts, but part one may already constitute a sufficient learning opportunity for some of you and so you may view part two as optional.

Implementing the Task

Launch

Before starting the task, you may wish to solicit some ideas or recall for students the structural characteristics of linear relationships versus exponential relationships. We also recommend allowing the students to familiarize themselves with the COVID-19 spread data itself before any equations are introduced. You might wish to ask questions like the following:

- Based on these first few weeks of COVID-19 in the United States, does the spread of the disease over time seem to be **linear** or **exponential**? Explain how the data fits or doesn't fit with what we know about linear relationships or exponential relationships.
- A person online says the spread is neither linear nor exponential. They say that the number of COVID-19 cases went up by 34 from March 6th to 7th but then only went up by 2 from March 7th to 8th. These changes in the number of cases are different, so it's

not linear, and the change from March 7th to 8th was smaller than before, so it's not exponential. What do you think?

Ideally, the students will accept that these data are exponential in nature, or at least decidedly non-linear. Although the table is not perfectly exponential, it does have many of the features of exponential growth and you should look for opportunities to highlight that COVID-19 spread is not just exponential because of the numbers in the table, it is exponential because of the nature of contagious disease. As more people become infected, they contribute to new opportunities to spread the disease. So there is not a constant influx of disease but rather a compounding spread of the disease. This point involves a connection between mathematical functions and health literacy in terms of understanding the risk of contagious diseases in a population.

When asked about linear versus exponential growth, some students may quickly state that it is exponential because they have heard this and simply know it is the correct answer, but in this case you should still look to probe a bit deeper. Point out that the data is not perfectly exponential, there are some ebbs and flows that are unavoidable with real-world data. The second bullet above can help to generate conversation about this point.

Once you feel that students understand the data table, you can provide the function in Part One of the task. You may wish to describe the two variables (*cases* and *days*) but then leave it to the students to make sense of the numerical components of the function. We have supplied a reasonable exponential function that you may use but if you have time and would like to engage students in a fuller experience of mathematical modeling, you may have them come up with their own exponential function for Part One (link to Spreadsheet More4U). If you choose to do that, we encourage the use of technological tools such as Desmos, which can greatly speed up the opportunity for students to check the accuracy of their draft functions.

Explore

We encourage you to give students opportunities to work individually and then with partners or groups on each item in Part One. As students work, you can monitor for clear connections between the function and the context, such as the notion that there were presumably a few initial cases of COVID-19 (14) and that there is a base greater than 1 (1.39), meaning that the cases will rise as each infected person contributes to an average of 1.39 newly infected people. You can also monitor for confusion between the function and the context. We describe some important ideas that might come up in the Students' Thinking section below. You may wish to discuss and summarize question #1 separately and then move them back into groups or work time to complete #2 and #3.

As students work on #2 and #3 in Part One, you can think about selecting and sequencing particular student responses. For #2, you might look for two different ways that students express the change from a constant term of 14 to a constant term of 7, but in both versions of the explanation they are highlighting that COVID-19 ended up basically with the same spread at the end of the month. For #3, we recommend selecting a student (or inserting this yourself if it does not come up) who mistakenly puts the 0.9 multiplier in front of the exponential term (i.e., $cases = 0.9 \cdot 1.39^{days} + 14$) and also a student who correctly applies the 0.9 to the base itself (i.e., $cases = (0.9 \cdot 1.39)^{days} + 14$). We discuss this idea further in the Students' Thinking section below.

Summarize

When discussing #1, you can call on students to put into their own words the connections between the function and the context. You can also make some connections to health literacy because, in epidemiology, the base of the exponential function (1.39) is very similar to the effective reproductive number. If it is greater than 1, the disease is spreading, and if it is less than 1, the disease is waning. For #1c, the students may have declared that the model would not be accurate beyond April 2020 but it would be good to solicit several reasons why it cannot be extrapolated perfectly (e.g., people may change their behaviors, eventually many people have already gotten the disease).

For #2 and #3, you can invite students to share their reasoning and the main idea will be that changing the initial cases, even by as much as 50%, does not have a drastic effect on the spread, but reducing the spread rate, even by as little as 10%, does have a drastic effect. We think it is worth explicitly discussing the difference between $cases = 0.9 \cdot 1.39^{days} + 14$, where the number of cases is reduced by 10%, and $cases = (0.9 \cdot 1.39)^{days} + 14$, where the actual spread rate is reduced by 10% (the latter is what is implied by the problem text). This is one of the most important opportunities to make connections between student responses, or to draw distinctions.

Finally, as with any modeling scenario, it is important to discuss with students the limitations of the model. In this case, an important one is to note that exponential growth cannot possibly continue upward unabated. An exponential curve can be a reasonable fit within a given time period, but then other constraints (e.g., there are only so many people on Earth, some will have already been infected, some have no contact with other people) come into play and start shaping the trend in different ways. The fact that events or behaviors change and the mathematical model breaks down does not mean it is invalid. This is an opportunity to emphasize that mathematical modeling is a dynamic process, there is always some amount of error, and a model should come with an understanding of when and where it is reasonable to use it.

Students' Thinking

During the launch, as students seek to understand the COVID-19 data itself, they may get confused between total cases and daily new cases. We have provided the daily *new* cases because this is often what is reported in the news and also for simplicity because we did not want to have to account for people recovering and thus no longer being infected. But focusing on new cases could cause confusion when thinking about what it would mean for a linear spread of a disease—a basic linear spread of a disease might mean that, say, 10 people are newly infected each day. But a linear relationship for the daily *new* cases would mean that 10 people are infected one day, then 20 the next, then 30 the next, and so forth (thus a linear growth of new cases is a quadratic growth of the cumulative cases). You should be aware of this distinction but we do not recommend addressing it in the task enactment unless it comes up.

Also during the launch, we encourage you to check with students about whether they think the data was *always* exponential during that month of spread or if they think it was really only exponential once it started to take off. Exponential curves are fairly flat at the beginning but they are still exponential, even before the big upswing is visible. This webcomic in Figure 1 may be useful in avoiding this misconception about exponential functions (*original on Twitter; PUBLISHER MAY NEED RIGHTS*).

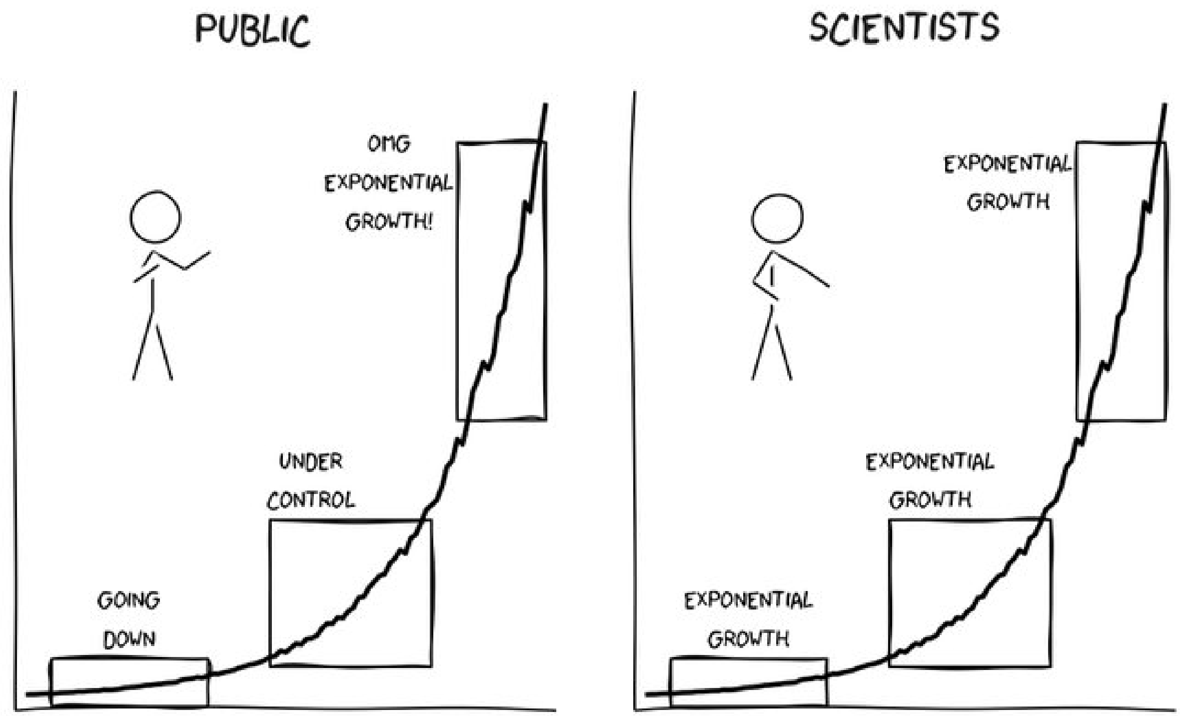


Figure 1. A webcomic from @vb_jens, posted on Twitter, that makes the point that exponential curves are exponential from the start.

In summarizing the ideas from the problem, it is important for students to realize that the base of the exponent is much more consequential than the constant term. And with regard to the base, note that, with regard to 10% protection from masks, it might be helpful to think about that as 90% of infectious spread is still happening. In the context of the problem, truly reducing the spread (so the 1.39 base becomes 1.25 and the exponential curve is bent noticeably downward) is very different from letting the spread continue just as dangerously and then merely reducing the number of cases to 90%. Masks actually reduce the transmission from person to person, so it affects the base of the exponent, not just the total output, and so masking would have a big impact already in the first month (see Figure 2).

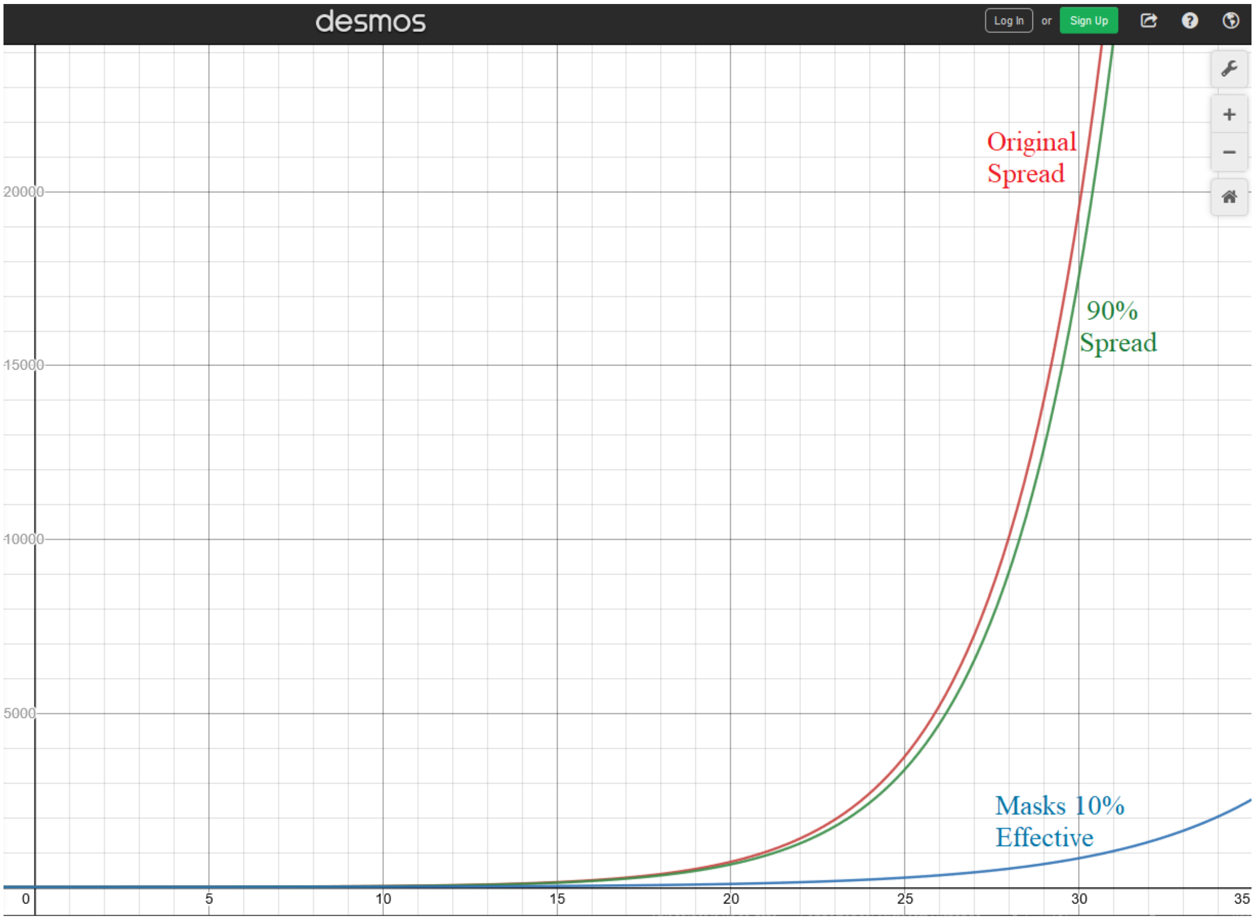


Figure 2. Reducing the base transmission rate is much different than merely reducing the total number of cases.

Part One of this task provides students with opportunities to think about an exponential function, and its various components, in the context of viral spread. But their thinking can be pushed further if they have an opportunity to create a second function. This creativity may also lead to deeper buy-in and perhaps a sense of responsibility about safety precautions related to COVID-19, which leads us to Part Two and some other ideas about extensions to this task.

Extensions

You may decide to end the task after Part One, but Part Two invites students to bring their own ideas to encode their own ideas into a mathematical model. And Part Two also touches on the mathematical standard of comparing and combining multiple mathematical functions. It is possible that political differences or tensions may arise because some safety recommendations have been politicized. We leave this to your judgment in terms of how to handle it with your students and in your local context. If time or resources aren't available, you might choose to use rough guesses for the effectiveness of certain ideas (so if a student says they support restaurants as takeout only, you could simply say, "Great, let's just imagine that that reduces the infection rate by say 30% and see if you can work that into your equation"). If time allows, you could ask them to justify the numbers they use ("Great idea, why don't you look online to see if you can get an estimate for how much of the spread of COVID-19 is from indoor dining"). This may lead to issues of source validity and trustworthiness, which connects directly to media literacy and ideas which also come into play in Task 3.

The final items in Part Two invite students to directly compare their revised model of initial COVID-19 spread with the original model and the actual data. They may notice that, because of the nature of exponential spread, things that are very similar at the start can nevertheless be quite different a month later. Functions can also be combined through subtraction to create a new function that shows the number of cases prevented by the students' hypothetical safety precautions. If you want to make this even more poignant, you may also wish to consider deaths avoided. To estimate the death rate, we used the total COVID-19 deaths in the United States as of November 2021 (757 thousand) divided by the total number of cases (46.7 Million), which is fairly close to other calculations of COVID-19's fatality risk.

For further extensions, one idea is to work similarly to this task but in a situation where the exponential spread is decaying rather than growing. For example, the months after the vaccines were introduced in the USA show a roughly exponential downward trend (until the delta variant, that is) and this mathematically corresponds to a situation where the base of the function (i.e., the effective reproductive rate) is less than 1 (see More4U data spreadsheet). Another key extension could be for those who wish to make calculus connections. We have been dealing with the daily new cases, but students may also want to tally up the total cases for the month, or the total number of cases prevented or lives saved from their safety measures. We encourage the use of spreadsheets and a discussion of how this is summing up each daily number along the function curve.

Task 2: Probabilities of Protection

Imagine a contagious disease that is spread through the air. This fictional disease is so contagious that each week, everyone has a 10% chance of catching the disease regardless of what they do.

1. After 3 weeks, what is the chance that you will have caught the disease?

Thankfully, people can do the following things to reduce their risk* of catching the disease, such as using a special air filter that reduces risk by 60%:

Action	**Reduction in Risk Level for the Person if they do the action perfectly for a full week**
Using a special air filter in your home	60%
Wearing a cloth mask whenever you are around other people	70%
Wearing two cloth masks whenever you are around other people	91%
Wearing a special medical mask whenever you are around other people	90%
Avoiding any gatherings of more than 3 people	40%

Staying more than 6 feet away from other people	50%
Wearing a nose plug all the time	20%
Breathing through your nose all the time except when you're talking	80%

**Assume that these actions and their risk reductions are independent of one another.*

2. Choose three of the actions that you would do to try to keep safe from the disease.

   Recall that your normal weekly risk is 10%. For each action, calculate your new weekly risk if you did just that one action.

   a. Action 1: ________________________ New weekly risk: ______%

   b. Action 2: ________________________ New weekly risk: ______%

   c. Action 3: ________________________ New weekly risk: ______%

3. Imagine that you did Action 1 for one week, then Action 2 for the next week, and Action 3 for the third week. What is the overall chance that you will catch the disease during those three weeks?

4. What would be your weekly risk if you did **all three** of the actions you selected in #2?

5. Imagine that you did all three of your actions for three weeks. What is the overall chance that you will catch the disease during those three weeks? How would you show or explain to someone else the difference in risk between doing nothing (see #1) and doing all three of these safety actions?

6. You see a post online that says the following:

   If you install the special air filter (60% protection) and also avoid gatherings of more than 3 people (40%), then you are 100% protected against the disease.

   Would you share this post? If yes, explain why. If not, what would you say in reply to that post?

Standards

- CCSSM Practice Standards
  - MP2 Reason abstractly and quantitatively.
  - MP3 Construct viable arguments and critique the reasoning of others.
- CCSSM Content
  - **7-SP.8** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
  - **7-RP.3** Use proportional relationships to solve multistep ratio and percent problems.
  - **HSS-CP.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
  - **HSS-MD.7** Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Situating the Task

This task is written to be appropriate for a 7th grade probability or percentage lesson but can also be adapted for use in a high school statistics or probability class. The manner in which students' represent their thinking might vary across the grade levels but, in both contexts, they can engage with the central ideas of compound probabilities and using percentage concepts to inform safety decisions, which connects with notions of health literacy. The main adaptation that we recommend for a high school setting is to discuss with students the issue of independent events. In the task, for simplicity, we invite students to assume the various safety measures are independent of one another, but high school students should be able to justify which actions are likely to be truly independent (e.g., using an air filter in the home and avoiding gatherings of 3 or more people) and which are probably dependent in some way (e.g., wearing a mask and also a nose plug is somewhat redundant).

This task can be completed by students individually, and there is an element of student choice as they decide which safety measures they want to focus on for their probability calculations. But this task can also be completed collaboratively or it can provide an opportunity for comparison and conversation after students have worked because they can look across their various choices. Some students may wish to strive for the safest choices possible, but the task does not specify that it has to be the safest choices; students may have various reasons for their choices (convenience or likelihood of carrying out the action successfully, not just the lower possible risk) and so the classroom norms should allow for respectful disagreements and variety in students' choices.

With regard to technology, the probabilities were chosen to be simple enough that students can work with them directly but calculators or spreadsheets are welcome if they are part of the classroom culture. It is helpful if organization and systematic presentations of mathematical work are emphasized because that will allow for the clearest comparisons of doing nothing, doing one thing, or doing multiple things to lower the risk of catching the disease. These comparisons will be crucial for students' comprehension of compound probabilities.

Implementing the Task

Launch

You can decide whether to connect this fictional disease to any real-life contagious diseases, particularly those that are airborne, but we think the task can work by simply proceeding with the simplified, fictional scenario and students can then make connections, implicitly or explicitly later on, to any real-life analogues. If you wish, you can have students give a name to this disease but you should only do so if you are confident that their naming tendencies will avoid any insensitive or offensive choices.

Before actually passing out the task and revealing to students the various safety actions, we recommend launching the task by using question #1 as a whole-class discussion. This will ensure that students are understanding the baseline situation with a 10% risk of infection each week (so a 90% chance of remaining uninfected the first week, an 81% chance of remaining uninfected after two weeks, and a 72.9% chance of remaining uninfected after three weeks or, equivalently, a 27.1% chance of getting infected). You can also establish with the class the flexibility of thinking about the risk of infection or the probability of remaining uninfected.

You can then introduce the list of safety actions. These actions, in reality, do reduce the risk of an airborne illness but the percentages listed are fictionalized for mathematical purposes. Two important points with this list are that (1) we are assuming perfect adherence to the safety action to get the given risk reduction (e.g., wearing a cloth mask correctly and consistently can yield a 70% reduction in risk, so down from 10% risk to only 3% risk, but if you only wear it sometimes then you'll only get some of the protection), and (2) we are assuming independence between the actions. However, as noted above, if you wish to address independent and conditional probability (HSS-CP.5) then you should make it a discussion point about which actions are likely independent and which are not independent.

Explore

Questions #2 and #3 can be worked in small groups, as these problems deal with safety actions being taken one at a time and the probability calculations are a bit more straightforward. As you monitor student work, look for the different choices students make about safety actions. Some may choose a cloth mask or a medical mask because of the relatively high level of protection they afford, but others may not like masks and so may opt for other actions that do not involve anything on their face (e.g., air filter, avoiding gatherings). Knowing the range of choices will inform your facilitation of a whole group discussion of these choices. You can also monitor their calculations to make sure they are appropriately reducing the risk. For example, when reducing a 10% original risk by 70%, this would leave only a 3% risk of infection but some students may think it is a 7% risk of infection (70% of 10%). In this case, you could intervene and ask the student, “If you are 70% *protected*, then how much risk is still leftover?”

In their work on question #3, you can look for students to calculate, based on their specific choices, a probability that is substantially lower than that calculated during the launch (question #1) due to the increased safety measures. Just as before we had a 27.1% chance of getting infected, so too should the students in #3 think about the cumulative probability of getting infected (or, conversely, of remaining uninfected) if they do their safety actions for one week of the three weeks.

Once students have completed their work on question #3, you may choose to do a brief whole-class check-in conversation or let them continue onward to questions #4–6. In this next set the students will be layering multiple simultaneous safety actions. Instead of tracking probabilities over multiple weeks, they will have to reduce the risk in a compounded way. As you monitor the work, you should, if possible, select some student work that involves students multiplying the reduction percentages (instead of the remaining risk percentages), which is incorrect but a common inclination. This error will be worthwhile to discuss with the class later and we illustrate it in the Students’ Thinking section below.

As they work on question #5, you may have to prompt students to go beyond a calculation to explicitly compare their new (low) risk level with the original risk level in #1. They should have calculated the new risk level and they should somehow represent it in terms of the baseline risk; there are multiple ways to do this, as described below in Students’ Thinking.

As students work on question #6, listen for student reasoning as to why they would not share the erroneous social media post. Students should do more than just disagree with the post; try to prompt them to think about a reply that they would send to the person. If some students think the post is correct—that 60% and then 40% reductions in risk yields a total of 100% reduction—it would be a good idea to address this while groups are still exploring, or if many groups have the same impression, wait to address it more globally during the summary.

Summarize

We recommend focusing the summary on three overarching ideas:

A. How did you calculate the cumulative risk over three weeks?
B. How did you calculate the compound risk when you had three overlapping safety actions?
C. How would you explain your reasoning about the virus and about the social media post to someone else?

For the cumulative risk over three weeks, students should be able to express that as more time goes by, there will be more of a chance of getting infected, but the additional risk that comes with each passing week can be reduced by taking safety actions.

For the compound risk reduction of three overlapping safety actions, the discussion will be different if you assumed independence (as we indicated in the original task) or if you allowed the students to consider possible dependencies (as might be relevant given the high school standards). In either case, the risk of infection is reduced each time a new safety action is taken. And like the cumulative risk over time, the students are multiplying probabilities together and have to do so in a thoughtful way.

The summary discussion should also include reasoning and argumentation. In real-life media, health risks are often not expressed as an absolute probability but as a relative probability. This is why it is important to help students be explicit about how the new (lower) risk compares to the original baseline risk of infection and reason multiplicatively about those relative risks. Say, for example, that students had chosen the safety actions of using an air filter at home, wearing a cloth mask, and staying 6 feet away from others. This means the weekly risk of infection has gone down from 10% to only 0.6%. Both of these are fairly low, but 10% is almost 17 times higher risk than 0.6%. After three weeks, the original risk of infection was 27.1% but the new risk would be only 1.79%, again, the former is more than 15 times higher risk than the latter. Finally, there may be an opportunity when discussion responses to question #6 is worth discussing as a way to allow students to critique incorrect reasoning. Many people will add percentages when in fact the way they interact is through multiplication. Situating this critique in the context of social media can also tie into media literacy, which is discussed in more detail in Task 3.

Students' Ways ofThinking

Students have choices in this task. Though they may feel pressure to choose the safest options, the goal here is not to find the safest set of actions (those would simply be the ones with the largest reductions in risk), the goal is to think about how probabilities accumulate over time and how they compound with multiple actions. Students should feel free to choose based on many variables including safety levels or personal preferences—both are valid. Regardless of their choice, a key is to think about a 60% reduction in risk as still leaving 40% of the original risk. Note that the 6-foot social distancing action has a 50% reduction in risk, which means a student could accidentally end up with the correct answer because the 50% reduction and the 50% remaining risk are the same.

A common error when students compute risk over three weeks is to add together the risks. For example, if they have a 4% risk of infection the first week, then 6% the second week, and 3% the third week, they may think there is a 13% chance of getting infected overall. This is not far off of the true answer (12.5%), but the reasoning is inappropriate. We have found that a good way to reveal the error is to use larger percents as a counterexample. If there was a 60% risk of getting infected each week, then the additive approach would yield a 180% chance of getting infected over three weeks, which is an impossible percent. Another error is to multiply the risk percentages rather than the probability of remaining uninfected. So using the same numbers as before, a student may say that there is a $(0.04)\cdot(0.06)\cdot(0.03) = 0.000072$ or 0.0072% chance of getting infected, but this does not make sense because you have more of a chance of getting infected over three weeks than you do of getting infected in just the first week, regardless of how safe you are.

With regard to the comparison of the new (lower) risk level with the three safety actions to the original risk level, students may report the two risk levels side by side (e.g., 1.79% and 27.1%) or they may simply say that the new risk level is a lot lower. You should encourage them to quantify this comparison in some way. They may do an additive comparison ("The risk is 25.31% lower") but this can be misleading because when someone hears "25.31% lower," then they may think of it as removing roughly one quarter of the risk, so 27.1% risk would become approximately 20.3%, but this is far from the students' real answer of 1.79%. For this reason, and because small numbers are usually communicated about in a proportional or relative sense, you should look to emphasize multiplicative comparisons (27.1% risk is about 15 times higher than 1.79% risk).

Extensions

To extend question #3, ask students what would happen if they changed the order of their actions. What if you did Action 3 first, then Action 2, then Action 1 in the third week? How does that compare to your risk over the three weeks if you did the safety actions in the original order? They should find that the resulting risk after three weeks will be the same either way, but it could be different after the first week or the second week along the way. How would they prioritize their actions based on what they learned?

To extend thinking on question #5, ask students what would happen after three weeks. Can they project out to four weeks or five weeks or *n* weeks? If they were engaged in discussions of which safety actions are most feasible, you could also ask them how long they think they could realistically keep up the safety action, and then try to calculate their risk level over that time period (e.g., it is feasible to keep their air filter running indefinitely but they may only be willing to avoid gatherings of people for two months).

Finally, this task was based on a fictional disease and fairly simple risk calculations. But if you are interested in real probabilities related to the COVID-19 pandemic, there are lesson plans and interactive apps from the COViD-TASER project that deal with comparative probabilities and also having students respond to misinterpretations on social media related to risk and probabilities. See https://www.covidtaser.com/lessons for more.

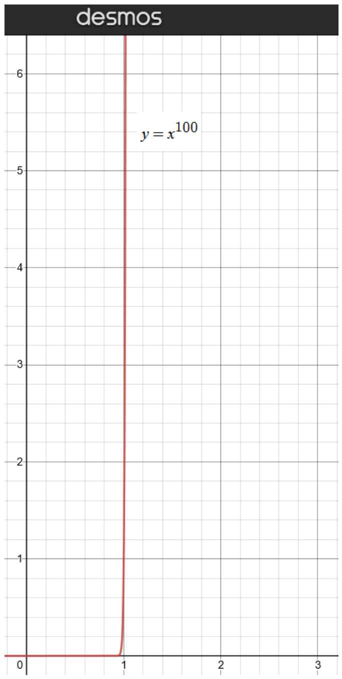
Task 3: Anti-Misinformation Mantra

"How do you know that is true?" **"Could you possibly be wrong?"**

These two questions are helpful in mathematics (and in life) for distinguishing false or faulty ideas from ideas that are well-justified. We can ask them of ourselves or of others and then think critically about the response.

1. Polynomials versus exponentials

a. User123 is on social media posting about their math class. They write the following about the polynomial functions and exponential functions they are learning about:



Polynomial functions can go up super fast. Look at this one with 100 in the exponent. It's basically straight up at x=1. There's no way that y=2^x *will be able to catch it. And this isn't even the steepest polynomial. Polynomials can even have degree 1 million! #PolynomialsForTheWin*

Choose one of the questions above, "How do you know that is true?" or "Could you possibly be wrong?", and imagine asking it of User123. What might User123 say in response to try to defend their post?

b. What do you think? Are you convinced by User123's post or by their possible response from the previous problem? Why, or why not?

2. Rational numbers versus irrational numbers

   a. UserABC is on social media posting about their math class. They write the following about the rational numbers and irrational numbers they are learning about:

   A rational number times another rational number is still a rational number. And an *irrational number times a rational number (not zero) is still an irrational number. But the funny thing is that an irrational number times another irrational number can be irrational or rational. #NumberGames*

Choose one of the questions above, “How do you know that is true?” or “Could you possibly be wrong?”, and imagine asking it of UserABC. What might UserABC say in response to try to defend their post?

b. What do you think? Are you convinced by UserABC’s post or by their possible response from the previous problem? Why, or why not?

Standards

- CSSM Process
  - MP3 Construct viable arguments and critique the reasoning of others.
  - MP6 Attend to precision.
- CCSSM Content
  - **HSF-LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
  - **HSNQ-RN.3** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Situating the Task

This task uses the two critical questions, “How do you know that is true?” or “Could you possibly be wrong?”, and ideally they would be part of a classroom culture where justifications are requested and provided regularly and where claims are often considered to their limits to see if they always hold up. This form of questioning should not come across as personal attacks but instead should be in the spirit of collective understanding and sense-making.

These critical questions can lead to careful mathematical thinking and precise mathematical communication, but they are also relevant in media literacy beyond mathematics. Media literacy is as important as ever because of the deluge of information and misinformation spread online, particularly on social media (Gleason & Von Gillern 2018). These particular questions, “How do you know that is true?” or “Could you possibly be wrong?”, have been found to be a successful way for students to inoculate themselves against misinformation (Jones-Jang et al., 2021). In other words, by regularly asking these questions, students are more likely to evaluate the validity of information and distinguish reliable information from questionable information. Researchers have found that this sort of critical thinking within media literacy can be taught, even in a single semester, and it can reduce students’ unwarranted beliefs (Dyer & Hall, 2019).

For this particular task, we incorporated the topics of high school functions and high school number and quantity, but you could use this same task template in other grade levels or other courses by swapping out the sample claims. These could be done early in the development of a concept (even to launch a big idea for a unit, possibly) or they could be done as conceptual review and formative assessment. You could also have students make their own claims and then invite discussion and debate as the claimant answers how they know it to be true or how they are able to rule out ever possibly being wrong.

Implementing the Task

Launch

There is a delicate balance during the launch between makjng sure that the students comprehend the claim being made by User123 in statement #1 and by UserABC in statement #2 without reteaching the material. Resist any urge to justify or over-explain the claims. That way, during the explore phase, the students can focus on thinking about justifications, possible defenses, and potential counterexamples to the claim.

You can also use the launch to clarify to the students the format in which you would like them to produce their answers. Based on your preference, they can talk it through and be ready to share their answers verbally or in writing, and the writing could be in the format of a school assignment or it could be informal as in a response to the social media thread.

Explore

Students can work individually or collaboratively on this task. As they engage with the thinking expressed by User123 or UserABC, some students may imagine User123 completely giving in and realizing the error of their ways as soon as they are asked a critical question. If you notice this happening, you might prompt for deeper thinking with, “I see that you had User123 change their mind right away. But what might cause this person to hold more tightly to their argument? What if this person really believed the original claim and wasn’t going to change their mind so easily? What do you think this person might say if they were going to try to defend it for a bit longer?” You can also let the students know that they don’t have to agree with User123 in part a, they just have to temporarily consider things from their point of view. Students will have a chance to express their own opinions in part b.

If students begin the task unsure about the long-term trends of polynomial growth versus exponential growth and are having difficulty seeing that exponential growth will eventually overcome, you can encourage them to use technology to consider some large numbers. This may, however, still be difficult if the students are keen on trying very large numbers. In this case, you might encourage them to simply consider whether it's *possible* that an exponential function could eventually surpass the polynomial function. Students may not be able to fully prove this during the explore phase but should express at least some doubt that, just because a polynomial function is higher at the start and seems to be going up steeply does not mean it will necessarily remain higher than an exponential function forever. To maintain some doubt in the face of a plausible but false claim is an important step, both in mathematics and in real life.

As students work through statement #2, you can monitor for students who agree with UserABC (which is correct), students who disagree with UserABC, and students who are unsure. The agreement will likely come from specific examples such as an irrational times an irrational yielding an irrational number (e.g., $\pi\sqrt{2}$) and an irrational times an irrational yielding a rational number (e.g., $\sqrt{2} \cdot \sqrt{2}$). Note these examples and select students to share them during the summary. If students are having difficulty thinking through the statement, you can encourage them to think about as many different irrational numbers as they can and then consider their products.

If you see various student opinions on UserABC's claim, we recommend sequencing them by starting with those who disagree with UserABC before hearing from students who have some doubt about the claim, and concluding with those who have a justification for agreeing with UserABC. This leads us to the summarize phase.

Summarize

Our hope is that this task will set up a rich discussion where justifications are sought and carefully considered. You can start by first asking students to share what they think User123 would say to the question of "How do you know this is true?". Because User123's claim is not true, you or the students should be able to find some weaknesses in the justification. For example, just because a polynomial curve looks nearly vertical does not mean that an exponential curve won't be even *more* vertical eventually, and just because a polynomial curve is higher than an exponential curve in the visible window does not mean that this will be true for all values in the range. In this way, the question of "could you possibly be wrong?" should help to prompt thinking that will reveal the limitations of User123's justifications. You can then conclude by asking students for their own opinions and justifications. See the section on Students' Ways of Thinking for some details and resources related to these ideas.

Your discussion of statement #2 can be similarly structured, with the central difference being that UserABC has a correct claim (actually, three correct claims—about rational times rational, irrational times rational, and irrational times irrational). You should still ask students about what they think the response would be to "could this ever be wrong?". There are infinitely many rational numbers and infinitely many irrational numbers, so it is impossible to check them all, but we can use general reasoning to draw a conclusion about all products of rational numbers and all products of an irrational and a rational. We can also find specific examples that prove UserABC's claim that an irrational times an irrational can end up irrational in some cases and rational in other cases. Two well-chosen products (e.g., $\sqrt{2} \cdot \sqrt{2}$ and $\pi\sqrt{2}$) are enough to justify this claim.

As noted above, we recommend sequencing this discussion from disagreement to agreement with UserABC. If you were not able to monitor and select particular student arguments from the explore phase, you could start this summary discussion by asking for a display of agreement or disagreement (e.g., thumbs up, thumbs down, or thumbs sideways) and then proceed by calling on the disagree-ers, then the doubters, and concluding with the agree-ers.

Students' Ways of Thinking

The polynomial in statement 1 is free to be any degree and it is being compared to the basic exponential function of $y = 2^x$. Students may be initially convinced by User123's post and graphic, but it is important for students to not get over-confident in this initial idea. The *x*-axis is a very long span and a lot can happen way out toward the right beyond our normal frame of reference. Our first goal is to make sure students maintain at least a little bit of doubt or uncertainty about the claim. Although a complete justification of the correct claim, that $y = 2^x$ will eventually have values that exceed any polynomial function, is possible, you may choose to focus on informal arguments from students. If students do not generate one, you may also present a more formal argument to the students, such as the one that is associated with the Illustrative Math task (https://tasks.illustrativemathematics.org/content-standards/tasks/367). This argument is built on a comparison of the ratio of the function at *x* to the function at *x*+1. For $y = 2^x$ this ratio is 2, no matter how large *x* is. But for a polynomial function, as *x* gets very large, the ratio will approach 1. Thus $y = 2^x$ eventually has a larger slope than the polynomial function and, given an infinite domain, will eventually produce larger values than the polynomial function.

When considering statement #2, students will probably accept without question the first claim that a rational number times a rational number is still rational. This fact probably does not have to be belabored, but in the spirit of critical thinking, it is wise to still quickly ask how we know that it's true. With regard to an irrational number times a (nonzero) rational number, any pair that we test will certainly work out to be irrational, but how do we know that we'll never be wrong? One way students can think about this is to imagine that they were wrong, that the product was a rational number ($p \cdot q = r$ with *p* irrational and *q* and *r* rational). But in this case, we could multiply that product (*r*) by the reciprocal of the rational factor (1/*q*) and that would yield another rational number (r/q) equal to the supposed irrational factor (p), meaning that we didn't really have an irrational number to begin with (p was supposed to be irrational but turned out to be rational). This proof by contradiction connects nicely to the idea that we want to think through the possibility that we could be wrong (i.e., what does it look like if the claim were false?).

As for UserABC's main claim, students will have to consider what happens when two irrational numbers are multiplied together. If the students are always multiplying two *distinct* irrational numbers together, then they may think that UserABC is incorrect because this always seems to yield another irrational number. There may also be a bit of intuition that the product of irrational numbers should be irrational. In the spirit of skepticism, it is good that the students are questioning UserABC's claim, but so too should they question their own initial ideas and wonder whether they could possibly be wrong. Indeed, squaring any square root of a non-square rational number will yield a rational product. At this point, with examples of an irrational product and a rational product being possible, it does validate UserABC's claim.

Extensions

To extend this task, we recommend that you apply these two critical questions, "How do you know that is true?" and "Could you possibly be wrong?", to additional claims. You could]provide them or ask students to generate them. If they find a definitive flaw in a claim, you can also extend the work by asking them to revise the claim so that it is true. This provides opportunities for further reasoning (SMP3) and attention to precision (SMP6).

As we noted above, these sorts of questions are not only helpful in mathematics, but can also be asked of claims made in everyday life, such as on social media. A recent study found that the primary reason people fall for misinformation is not because they were predisposed to believe the misinformation but rather because they simply did not make the effort to ask critical questions about the sources or the conclusions (Pennycook and Rand, 2019). If we help instill in students the habit of asking how people arrived at their conclusions or whether it's possible that there might be another side to the story, then they might be at least partially inoculated against the faulty information that often runs rampant.

In that spirit, we have two additional extensions. One can be found in the More4U resources for this chapter and it includes a few real-life examples of mathematical or statistical claims being made, some with faulty reasoning behind them. Another is that you can explicitly talk with students about how asking the questions "How do you know that is true?" and "Could you possibly be wrong?" can reduce the spread of misinformation. Just as Task 1 in this chapter showed how even modest safety actions can drastically reduce the spread of a contagious disease, you could work with students to mathematically model a piece of misinformation going viral (maybe it is shared by 20% of people who see it and each person shares it with an average of 10 people) and then you could compare that viral spread with what would happen if people asked the critical questions (these questions don't stop all misinformation but they might, for the sake of argument, reduce by 30% a person's likelihood of spreading it). As in Task #1 with the spread of disease, we can also "bend the curve" of the spread of misinformation with even modest efforts to think critically.

Acknowledgements

This work was supported by a grant from the National Science Foundation (award #2010312). Any opinions, findings, and conclusions expressed here are those of the authors and do not necessarily reflect the views of the NSF.

References

Abaluck, Jason, Laura H. Kwong, Ashley Styczynski, Ashraful Haque, Md Kabir, Ellen Bates-Jeffries, Emily Crawford et al. "The Impact of Community Masking on COVID-19: A Cluster Randomized Trial in Bangladesh." (2021). *Cowles Foundation Discussion Papers*. 2642. https://elischolar.library.yale.edu/cowles-discussion-paper-series/2642

Aufderheide, Patricia. *Media Literacy.* A Report of the National Leadership Conference on Media Literacy. 1993. https://eric.ed.gov/?id=ED365294

Chiucarello. https://ctsciencecenter.org/blog/take-10-covid-19-a-teachable-moment/

Dyer, Kathleen D., and Raymond E. Hall. "Effect of Critical Thinking Education on Epistemically Unwarranted Beliefs in College Students." *Research in Higher Education* 60 (2019): 293-314.

Gleason, Benjamin, and Sam Von Gillern. "Digital citizenship with social media: Participatory practices of teaching and learning in secondary education." *Journal of Educational Technology & Society* 21, no. 1 (2018): 200-212.

Howard, Jeremy, Austin Huang, Zhiyuan Li, et al. (2021). An evidence review of face masks against COVID-19. *Proceedings of the National Academy of Sciences of the United States of America, 118*(4), e2014564118. https://www.pnas.org/content/118/4/e2014564118

Jones-Jang, S. Mo, Tara Mortensen, and Jingjing Liu. "Does media literacy help identification of fake news? Information literacy helps, but other literacies don't." *American Behavioral Scientist* 65, no. 2 (2021): 371-388.

Koestler, Courtney, Mathew D. Felton, Kristen N. Bieda, and Samuel Otten. *Connecting the NCTM Process Standards and the CCSSM Practices*. Reston, VA: National Council of Teachers of Mathematics, 2013.

Kramer, Jillian. (2021, July 30). Struggling to assess pandemic risks? You're not alone. *National Geographic*. https://www.nationalgeographic.com/science/article/struggling-to-assess-pandemic-risks-youre-not-alone

Li, Tom, Yan Liu, Man Li, Xiaoning Qian, and Susie Y. Dai. "Mask or no mask for COVID-19: A public health and market study." *PloS one* 15, no. 8 (2020): e0237691.

Parks, Miles. (2021, March 25). Few facts, millions of clicks: Fearmongering vaccine stories go viral online. *NPR*. https://www.npr.org/2021/03/25/980035707/lying-through-truth-misleading-facts-fuel-vaccine-misinformation

Pennycook, Gordon, and David G. Rand. "Lazy, Not Biased: Susceptibility to Partisan Fake News Is Better Explained by Lack of Reasoning Than by Motivated Reasoning." *Cognition* 188 (2019): 39-50.

Shead, S. (2020, July 28). Facebook, Twitter, and YouTube pull 'false' coronavirus video after it goes viral. *CNBC*. https://www.cnbc.com/2020/07/28/facebook-twitter-youtube-pull-false-coronavirus-video-after-it-goes-viral.html

Smith, Margaret S., and Mary Kay Stein. *5 Practices for Orchestrating Productive Mathematics Discussions*. Reston, VA: National Council of Teachers of Mathematics, 2011.

Srinivas, S. (2020, March). How maths helps us battle the spread of infectious disease. *India Bioscience*. https://indiabioscience.org/columns/general-science/how-maths-helps-us-battle-the-spread-of-infectious-diseases

Stevens, Harry & John Muyskens. (2020, May). Disease modelers are wary of reopening the country. Here's how they arrive at their verdict. *Washington Post*. https://www.washingtonpost.com/graphics/2020/health/disease-modeling-coronavirus-cases-reopening/

Young-Saver, Dashiell. (2021, May 14). The math of ending the pandemic: Exponential growth and decay. *NY Times*. https://www.nytimes.com/2021/05/14/learning/the-math-of-ending-the-pandemic-exponential-growth-and-decay.html