BACKWARD TRANSFER EFFECTS ON ACTION AND PROCESS VIEWS OF FUNCTIONS

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Backward Transfer

- Definition: “The influence that new learning [about a new topic] has on prior ways of reasoning [about a previous topic]” (Hohensee, 2014)
- Influences = productive or unproductive (Hohensee, 2014; Jukić & Dahl, 2012; Macgregor & Stacey, 1997; Van Dooren et al., 2004)
- Constructs similar to Backward Transfer (BT):
  - *Retrospective Transfer* (Marton, 2005)
  - *Met-Afters* (Jukić & Dahl, 2012; Lima & Tall, 2008)
- Linguistics Research (Cook, 2003)
  - Bilinguals (L1 ← L2)

Action vs Process Ways of Reasoning about Functions and Backward Transfer

- Two ways of reasoning about functions - as *actions* or as *processes* (Breidenbach, Dubinsky, Hawks, & Nichols, 1992)
- **As Actions**: reasoning about functions as "repeatable physical or mental manipulation that transforms objects…to obtain objects" (p. 249)
- **As Processes**: reasoning about functions as a "complete activity beginning with objects of some kind, doing something to these objects, and obtaining new objects as a result of what was done" (p. 249)
- **Open question:** Whether ways of reasoning about functions as actions or processes are receptive/susceptible to BT influences?

Linear and Quadratic Functions and Backward Transfer

- Beginning levels of Algebra: linear functions prior to quadratic functions (Movshovitz-Hadar, 1993)
- Linear Functions → Quadratic Functions sequence = potential context for BT
- **Open questions:** What the conditions would need to be for quadratic functions instruction to influence students’ prior ways of reasoning about linear functions? What would be the nature of those influences?

Different Instructional Contexts and Backward Transfer

- Context 1: Regular algebra classroom and teacher, quadratic functions unit taught in the typical way, teacher not explicitly attending to students' prior ways of reasoning about linear functions
- Context 2: Experimental instructional setting, instructor is a Math Ed researcher, quadratic functions unit taught in a way that tries to influence prior ways of reasoning about linear functions by promoting covariational reasoning
- **Open question:** What kinds of BT influences occur in different instructional contexts?

Research Question

- Many open questions
- How do changes in students' prior ways of reasoning about linear functions as actions or processes compare between students who participate in an instructional unit on quadratic functions in a regular algebra classroom setting, and students who participate in an instructional unit on quadratic functions in an experimental setting that promotes covariational reasoning?
- Exploratory Research/Hypothesis Development Stage (Sloane, 2008)

Methods

- Setting
  - Phase 1: students learned from their regular teacher in their regular classroom
  - Phase 2: students learned in experimental instructional setting taught by a mathematics education researcher at the local university
- Participants
  - Phase 1: 57 students recruited from two integrated mathematics classrooms at two different Mid-Atlantic urban majority-minority public schools
  - Phase 2: 18 students recruited from a Mid-Atlantic organization that prepares low-income Black and Latino students for college

Methods

- Instructional Foci for Units on Quadratic Functions

Instructional Foci	**Phase 1**	**Phase 2**
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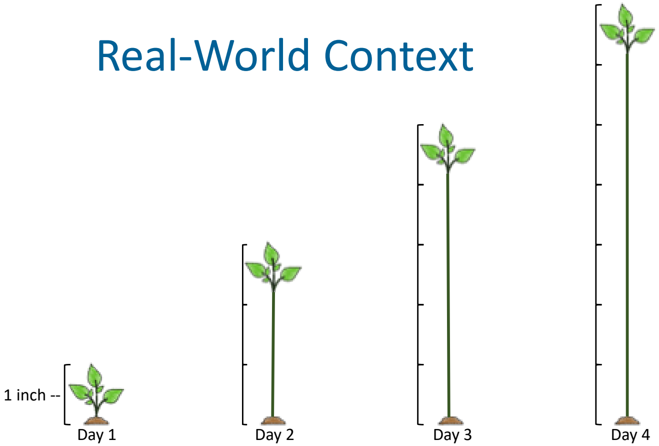
Methods

- Exploratory Research
- Data Collection
  - Pre/post written assessments, approx. 45 min each, all questions about linear functions, administered before and after the quadratic functions unit
  - Pre/post individual interviews, conducted with a subset of students
  - All quadratic functions lessons observed and video/audio recorded

Methods

Real-World Context



Three-Part Problem

a) Explain in words how to find the height of the plant on day 17.

b) Can you find the day the plant was measured if you were given the height? If yes, explain how. If no, explain why not.

c) You have to leave the plant in your office over the weekend. You did not measure the plant for 2.5 days. The plant grows at the same rate the whole time. How much did the plant grow in the 2.5 days you were gone? Show any work that helped you decide.

Methods

- Data Analysis - Number of responses coded in each phase
  - Phase 1
    - Coded 130 total responses (i.e., 57×3 minus 41 *did not attempt responses* or *insufficient info responses*)
  - Phase 2
    - Coded 53 total responses (i.e., 18×3 minus 1 *did not attempt response*)

Methods

- Data Analysis – development of codes
  - Phase 1:
    - First researcher - developed initial codes from pre/post-interviews
    - Research team - multiple rounds of coding of all pre/post-assessments to refine codes
    - All pre/post-assessments recoded once codes stabilized
  - Phase 2:
    - Use Phase 1 codes to code Phase 2 pre/post-assessments

Results

- Overview comparison between Phase 1 and Phase 2

Table 1

Students who changed their ways of reasoning pre- to post-assessment

	Change in Reasoning	**No Change in Reasoning**
Phase 1 (*N*=57)	34 (60%)	23 (40%)
Phase 2 (*N*=18)	10 (56%)	8 (44%)

Results

- Overview comparison between Phase 1 and Phase 2

Table 2

Problem parts indicating change ways of reasoning pre- to post-assessment

	Changed	**No Change**
Phase 1 (*N*=130)	65 (50%)	65 (50%)
Phase 2 (*N*=53)	15 (28%)	38 (72%)

Results

- Changes in ways of reasoning on Part (b) about whether it was possible or not to find the independent variable given a value for the dependent variable

Table 3

Students who changed their reasoning on Part (b)

Student	**Pre-Assessment**	**Post-Assessment**
Reece	Said it *was not* possible	Said it *was* possible
Jason	Said it *was* possible	Said it *was not* possible

Reece: Not Possible ⟶ Possible

Jason: Possible ⟶ Not Possible

Pre-Assessment

Reece:

b) Can you find the hour the rain water was measured if given the height? If yes, explain how. If no, explain why not.

No unless that start time is given, other than that no because if just given height we do not know how long it has been raining. If the equation is y=2x & we don't know any x the answer will be inaccurate. (?)

Jason:

b) Can you find the day the plant was measured if you were given the height? If yes, explain how. If no, explain why not.

Yes cause the plant the rate of change of two inches a day we would just work backwords from day 4 subtract -2 from each day till day day of 1 inch

Post-Assessment

Reece:

b) Can you find the day the plant was measured if you were given the height? If yes, explain how. If no, explain why not.

Ex: $33 = 2x - 1$
$+1 \quad\quad +1$
$34 = 2x$
$\frac{34}{2} = \frac{2x}{2}$
$17 = x$

Yes you can find the day if just the height was given by simply solving the equation & finding x

Jason:

b) Can you find the hour the rain water was measured if given the height? If yes, explain how. If no, explain why not.

No could only be estimated or given in the situation the hour would have to start somewhere form a rate of change

Results

- Changes in ways of reasoning on Part (b) about whether it was possible or not to find the independent variable given a value for the dependent variable

Table 4

Phase 1 and 2 comparisons of ways of reasoning on Part (b)

	No to Yes	**Yes to No**	**Yes to Yes**	**No to No**
Phase 1 (n=35)	5 (14%)	4 (11%)	24 (69%)	2 (6%)
Phase 2 (n=18)	0 (0%)	1 (6%)	17 (94%)	0 (0%)

Connection to Action vs Process Reasoning

- "When the [student] has a process conception, he or she will be able, for example, to combine it with other processes, or <u>*even reverse it*</u>" (Breidenbach et al., 1992, p. 251)
- We interpreted *yes it's possible* to reverse the linear function as reasoning about linear functions more as a <u>process</u>
- We interpreted *no it's not possible* to reverse the linear function as reasoning about linear functions more as an <u>action</u>

Results

- Changes in ways of reasoning on Part (c) about finding the change in the dependent variable over a general interval of the independent variable

Table 5

Students who changed their reasoning on Part (c)

Student	**Pre-Assessment**	**Post-Assessment**
Kelly	Reason with a general interval	Reason with a specific value
Yedira	Reason with a specific value	Reason with a general interval

Kelly: General → Specific

c) You have to leave the plant in your office over the weekend. You did not measure the plant for 2.5 days. The plant grows at the same rate the whole time. How much did the plant grow in the 2.5 days you were gone? Show any work that helped you decide.

It grew 5 inches

1 day = 2 in
1 day = 2 in
half day = 1 in
= 5

Yedira: Specific → General

c) If you go away on a short trip of 3.75 days and leave your friend with instructions to care for the plant exactly as you do, how much can you expect the plant to grow in the 3.75 days you are gone? Assume the plant continues to grow in the same way.

Day 4 → 5.8
+
3 days
+1.4
1.4
1.4
→ 10.0

.75 × 1.4
1.05
+ 10.6
11.05

Pre-Assessment

Post-Assessment

c) You fall asleep while watching TV. You did not measure the rain water for 3.5 hours. It rained the whole time at the same rate. How much rainwater was collected during the 3.5 hours that you were sleeping? Show any work that helped you decide.

21cm

hour 4	5	6	7	7.5
14 +2	16 +2	18 +2	20 +1	21

c) If you go away on a short trip and leave your friend with instructions to care for the plant exactly as you do. You will be gone for 4 full days and six-tenths of an additional day. How much can you expect the plant to grow in the 4 and 6/10 days you are gone? Assume the plant continues to grow in the same way.

6/10 → 0.6

1.75 × 4
7 · 0.6
4.2

You can expect the plant to grow 4.2 inches more

Results

- Changes in ways of reasoning on Part (c) about finding the change in the dependent variable over a general interval of the independent variable

Table 6

Phase 1 and 2 comparisons of changes in ways of reasoning on Part (c)

	General to Specific	Specific to General
Phase 1	11	11
Phase 2	0	8

Connection to Action vs Process Reasoning

- "In process responses the input, transformation, and output were present, integrated *and fairly general*" (Breidenbach et al., 1992, p. 251)
- We interpreted *a general solution* on Part (c) as reasoning about linear functions more as a process
- We interpreted *a specific solution* on Part (c) as reasoning about linear functions more as an action

Discussion

- Summary of Results
  - Phase 1 had more changes in ways of reasoning about linear functions than Phase 2
  - One type of change observed in Phase 1, not observed in Phase 2
  - One type of change observed across Phase 1/Phase 2
    Phase 1 change in two directions
    Phase 2 change in only one direction

Discussion

- Significance of Results
  - Informs Hypothesis Development about Backward Transfer
    - BT can appear as an unintended influence
    - BT can move learners ways of reasoning about functions
      action → process action ← process
  - Generates new ideas for teaching quadratic functions
    - Emphasize covariational reasoning during quadratics instruction

Discussion

- Implications
  - Curriculum developers – emphasize covariational reasoning during quadratic functions instruction, to influence learners action → process
  - Make teachers aware of BT
- Future directions for ongoing research
  - Working with teachers

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