

# Backward Transfer Effects When Learning About Quadratic Functions

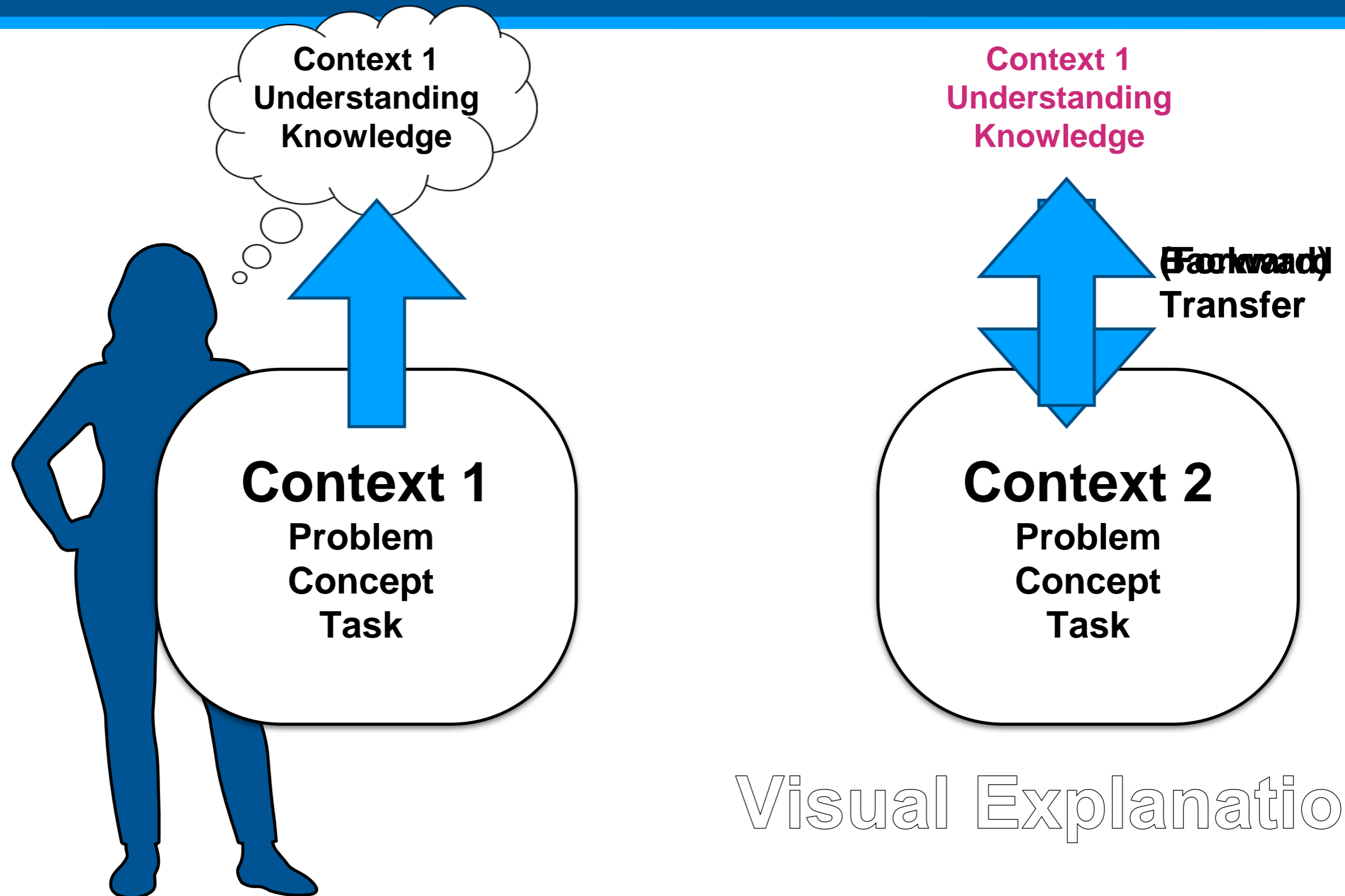
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DRL 1651571



# What is Backward Transfer?



Visual Explanation

# Theoretical Grounding for Backward Transfer

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- We know there is a relationship between prior knowledge and new learning (e.g., Hiebert & Carpenter, 1992)
- “Learning a domain of elementary mathematics or science may entail changes of massive scope...creating *very large ripple effects through the system*” (Smith, diSessa, & Roschelle, 1993, p. 148, italics added)

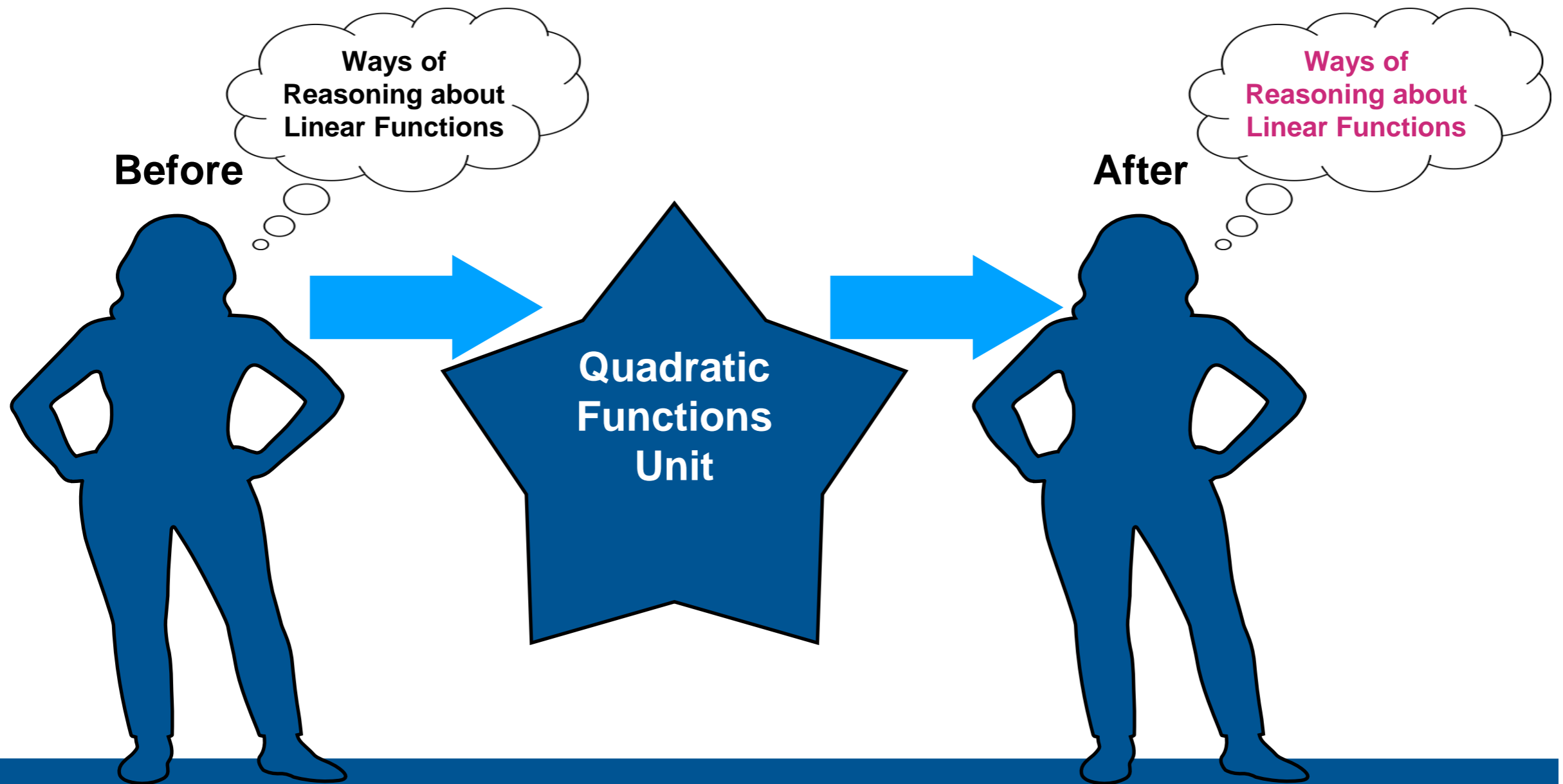
# Empirical Evidence of Backward Transfer Effects

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# Research Question

What kinds of changes in Algebra students' previously-established ways of reasoning about linear functions are observed after students complete a quadratic functions unit with their regular mathematics teacher?



# Methods

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## Participants and Setting

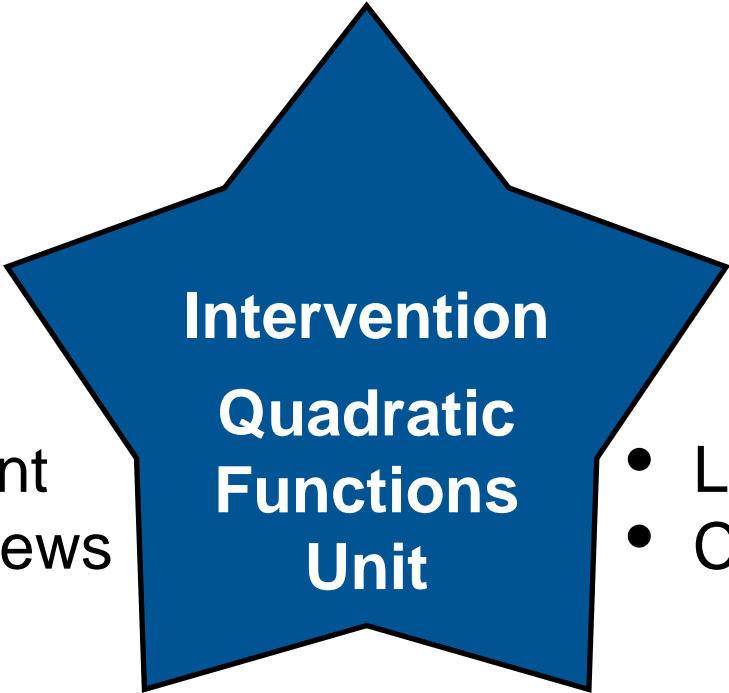
- 9th and 10th Grade Algebra Classes
- Teachers = +8 years experience
- Quadratic Functions curricula focus on properties of graphs and symbol manipulation

**Before**



- Linear Function Pre-Assessment
- Clinical Semi-Structured Interviews

**Intervention  
Quadratic  
Functions  
Unit**



**After**



- Linear Function Post-Assessment
  - Clinical Semi-Structured Interviews
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# Methods

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## Initial Analysis

- Analysis of three students' assessments and interviews
  - Descriptive narratives for each response for each student
  - Coded each response using “partway between the a priori and inductive qualitative approaches” (Miles & Huberman, 1994, p. 61)
  - Identified themes
  - Presented themes to research team as a preliminary check
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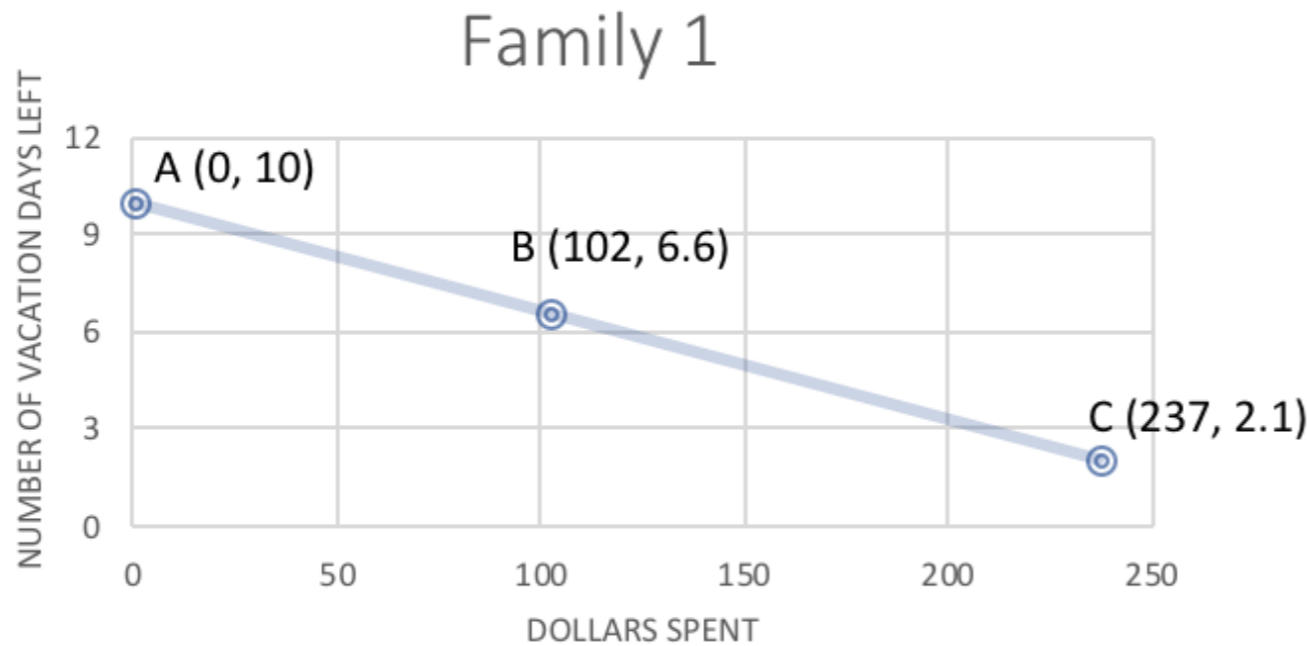
# Results

## Changes in Ways of Reasoning about Linear Functions

### 1. Reasoning with and without Changes in Quantities

**Phillip**

Before Quadratics



$$\begin{array}{r} 10 \\ + 6.6 \\ \hline 16.6 \uparrow = 17 \end{array}$$

$$17 \sqrt{102} = 6$$

$$\begin{array}{r} 6.6 \\ + 2.1 \\ \hline 8.7 \uparrow = 9 \end{array}$$

$$9 \sqrt{339} =$$

$$37.666666 \uparrow$$

es Family 1 plan to spend money at the same rate between points B and C.  
 No, family 1 will spend more money between points B and C.  
 17 has rounded. That's about \$60

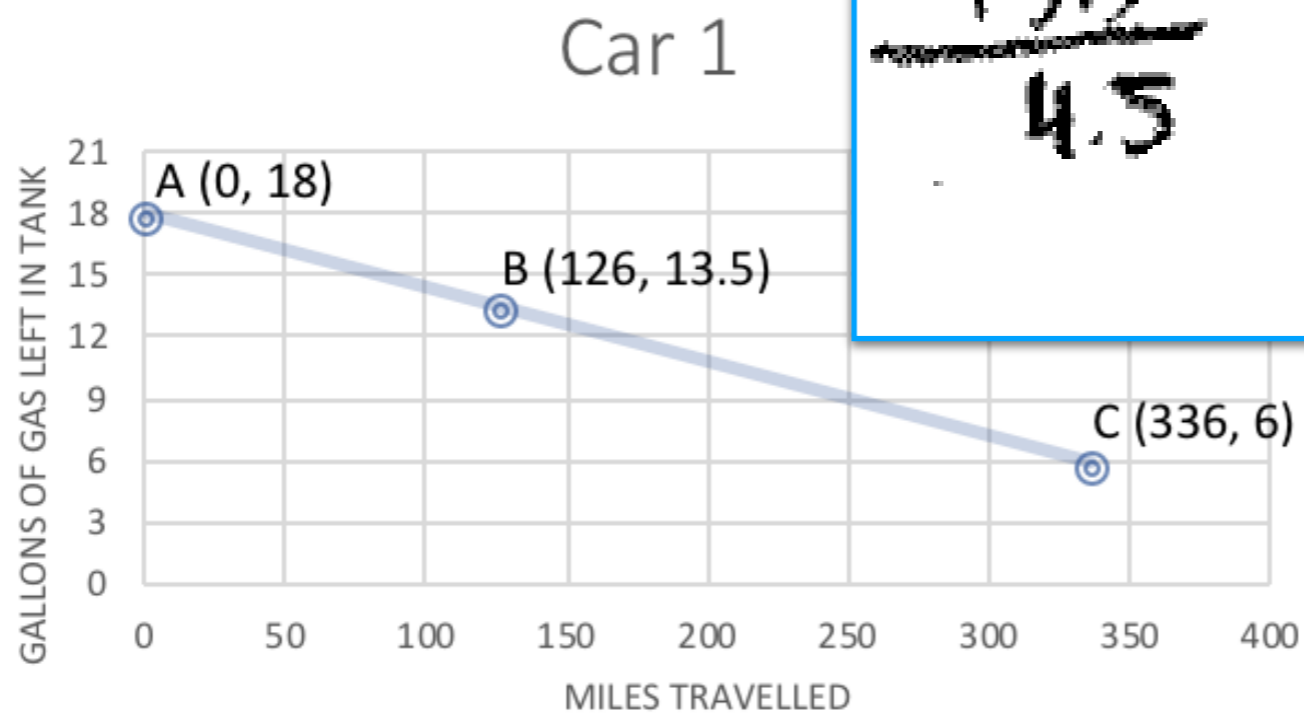


# Results

## Changes in Ways of Reasoning about Linear Functions

1. Reasoning with and without Changes in Quantities

Phillip  
After Quadratics



$$\frac{18}{13.5} = \frac{4.5}{4.5}$$

$$126 \div 4.5 = 28$$

$$\frac{18}{6} = \frac{12}{12}$$

$$336 \div 12 = 28$$

Car 1 does use gas at the same rate.

# Results

## Changes in Ways of Reasoning about Linear Functions

### 2. Correspondence View vs Covariational View of Functions

**Alex**

Before Quadratics

Hours Worked	Money Earned
3	\$55.50
7	\$129.50
13	\$240.50

Handwritten calculations showing a shift from a correspondence view to a covariational view. The left side shows a correspondence view where the rate is calculated for each point:  $3/55.50 = .054$ ,  $7/129.50 = .054$ , and  $13/240.50 = .054$ . The right side shows a covariational view where the rate is calculated for the entire set of data:  $55.50/3 = 18.5$ ,  $129.50/7 = 18.5$ , and  $240.50/13 = 18.5$ . A large 'X' is drawn over the correspondence view calculations, indicating they were discarded in favor of the covariational view.

$$40 * 18.50 = 740$$

The employee will earn \$740 after working 40 hours

# Results

## Changes in Ways of Reasoning about Linear Functions

### 2. Correspondence View vs Covariational View of Functions

Alex

After Quadratics

Additional MB of Data Used	Extra Fee Charged
15	\$10.50
27	\$18.90
33	\$23.10
36	

12 <  
6 <  
3 <

> 8.4  
> 4.2  
> 2.1

Handwritten calculations:

$$\begin{array}{r} 36 \\ - 30 \\ \hline 6 \end{array}$$
$$\begin{array}{r} 4.2 \\ - 2 \\ \hline 2.2 \end{array}$$
$$\begin{array}{r} 23.10 \\ + 2.10 \\ \hline 25.20 \end{array}$$

The extra fee is 25.10

# Summary

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## Answering the Research Question

- Quadratic functions learning activities can unintentionally influence students' ways of reason about linear functions.
- Looking at students reasoning about changes in quantities and the covariational vs correspondence view of functions seems promising.
- Changes in reasoning may be unintentionally productive. Changes in reasoning may also occur that are not more or less productive.

## Implications for Practice

- Teachers could provide students opportunities after quadratics lessons to reason in linear contexts with changes in quantities.
  - Teachers could emphasize the correspondence view during quadratics lessons so that they don't lose sight of it.
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# Extra Slides

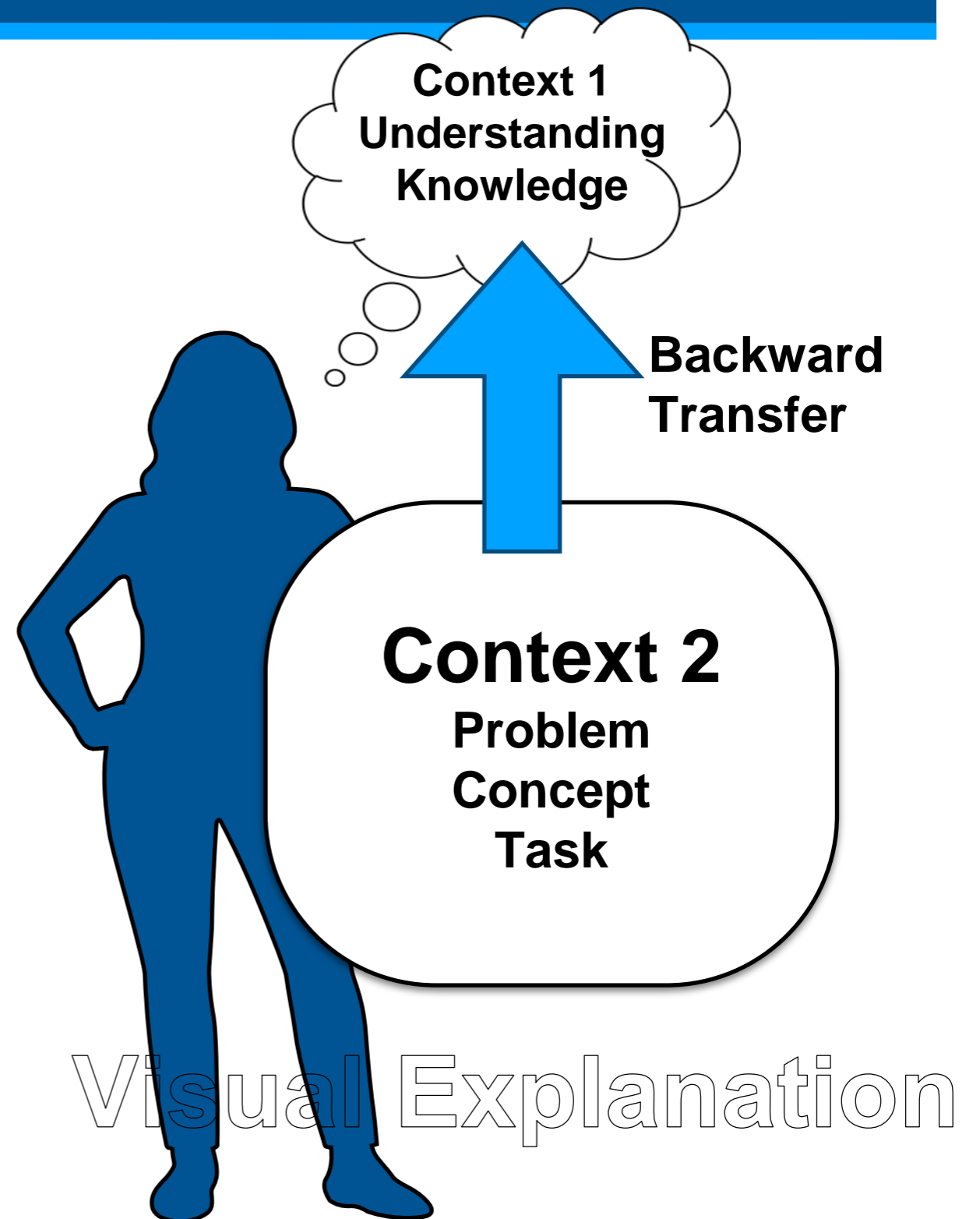
Participants	High School A	Teacher 1	9th Grade Classroom 1: $N_1=9$
		Teacher 2*	10th Grade Classroom 2: $N_2=27$
	High School B	Teacher 3	9th Grade Classroom 3: $N_3=18$
		Teacher 4	10th Grade Classroom 4: $N_4=24$

**$N_T=81$**

# Extra Slides

## Definition

Backward transfer is “the influence that constructing and subsequently generalizing new knowledge has on one’s ways of reasoning about related mathematical concepts that one has encountered previously” (Hohensee, 2014)

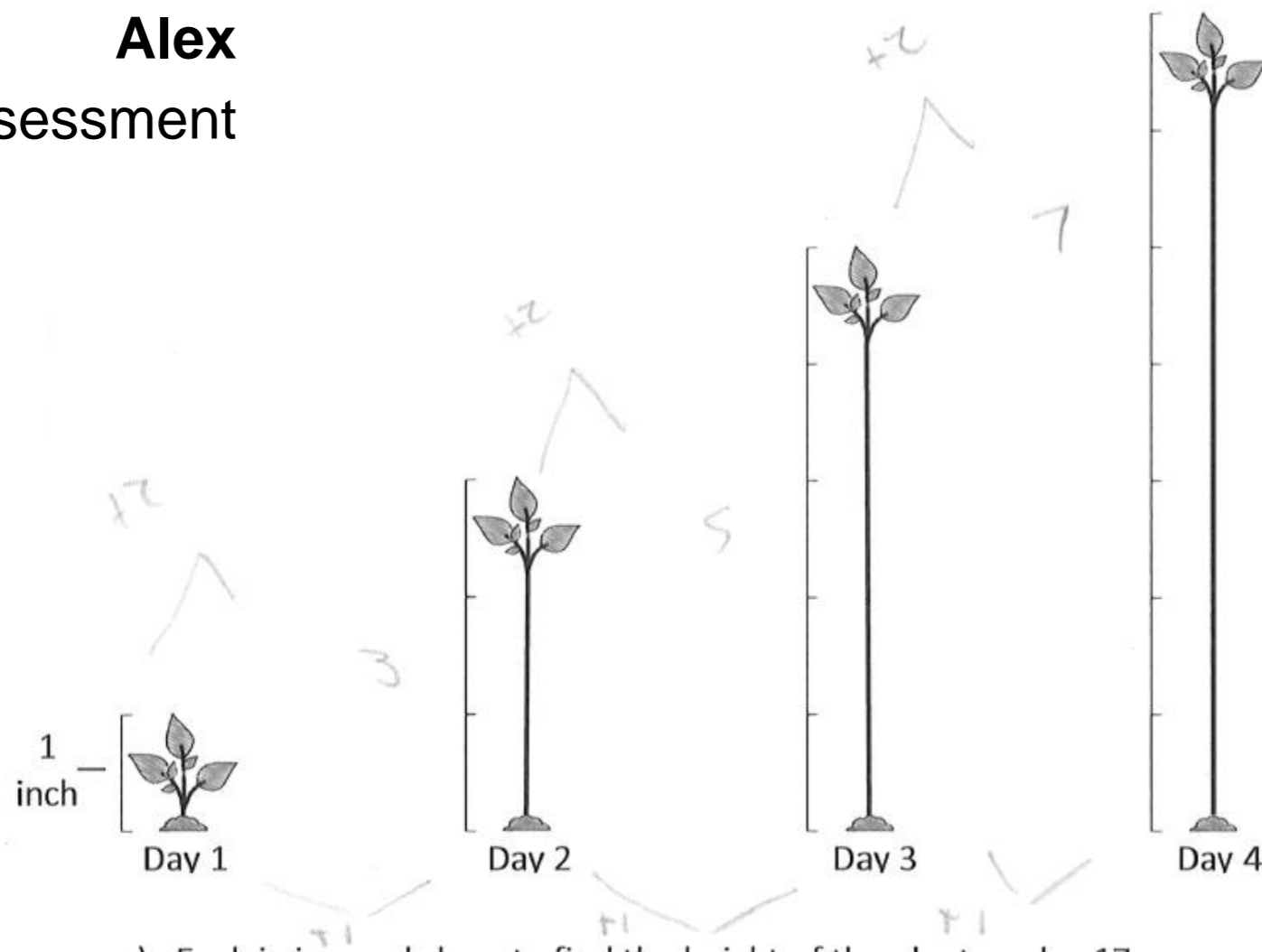


# Extra Slides

## Changes in Ways of Reasoning about Linear Functions

### 3. Additive vs Multiplicative Reasoning

Alex  
Pre-Assessment

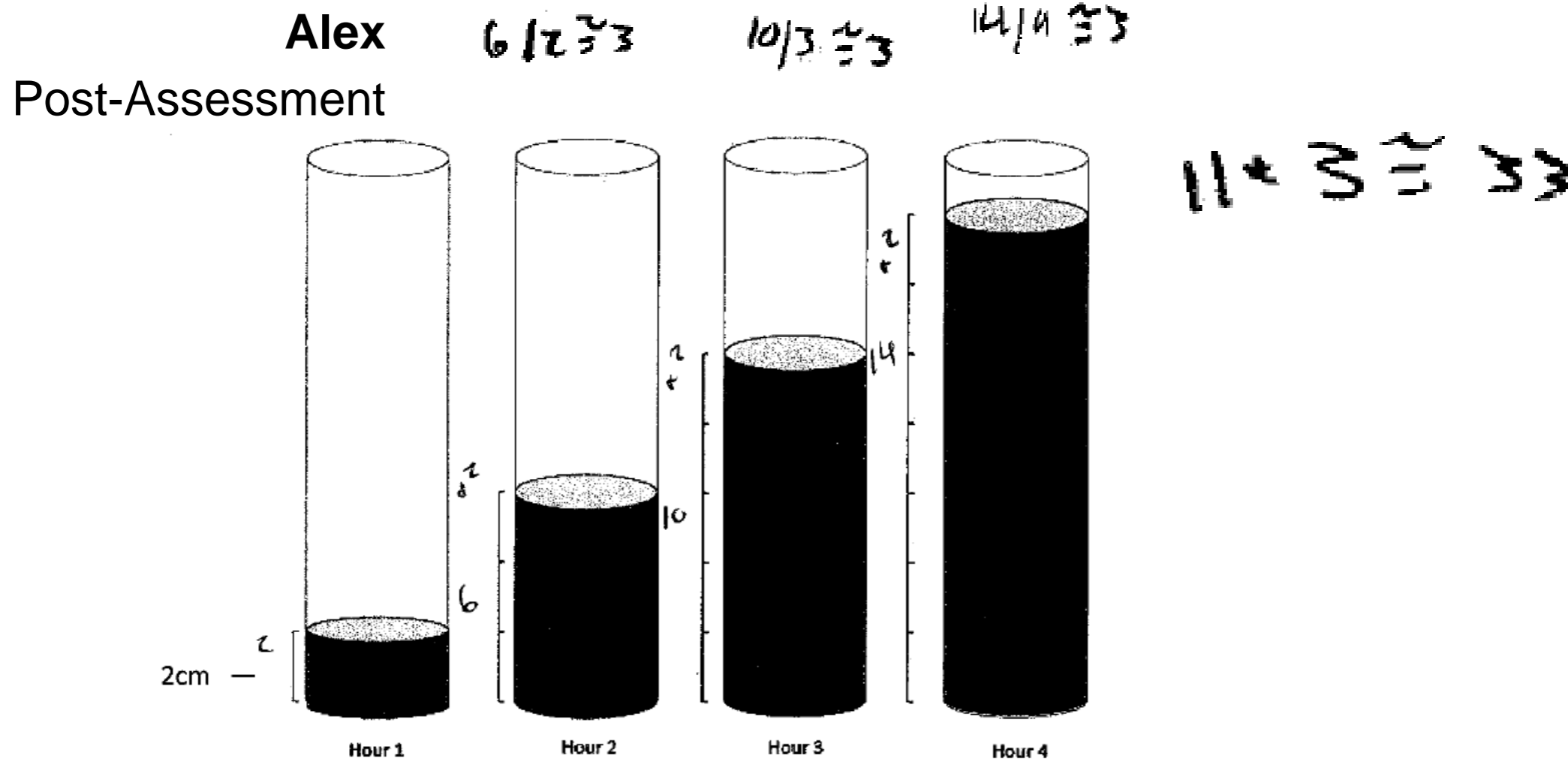


1	1
2	3
3	5
4	7
5	9
6	11
7	13
8	15
9	17
10	19
11	21
12	23
13	25
14	27
15	29
16	31
17	33

# Extra Slides

## Changes in Ways of Reasoning about Linear Functions

### 3. Additive vs Multiplicative Reasoning





# References

- Hiebert, J. & Carpenter, T. P. (1992). *Learning and teaching with understanding*. New York: Macmillan Publishing Co., Inc.
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- Hohensee, C. (2016). Hohensee, C. (2016a). Teachers' awareness of the relationship between prior knowledge and new learning. *Journal for Research in Mathematics Education*, 47(1), 16-26.
- Miles, M., & Huberman, M. (1994). *Qualitative data analysis*. Thousand Oaks, CA: Sage Publications.
- Smith, J. P., diSessa, A. A., & Roschelle, J. (1993). Misconceptions reconceived: A constructivist analysis of knowledge in transition. *The Journal of the Learning Sciences*, 3(2), 115-163.