## Penn GSE

INTRODUCTION
Number lines can be a powerful tool to help children make sense of addition and subtraction and develop number sense, understanding of magnitude, and flexible and efficient strategies, While number lines are used in many K-2 math program materials, explicit attention is rar given tributes of number lines so that students can attributes of number ines so that students use this moder with understanding (e.g., understanding that quantity is represented as a length or distance rather than discrete quantities)

Where does 19 go on the number line?

> result of counting on from 5 , but not in terms of magnitude in relation to the quantity of 5 .

## METHODS

We are in the third year of a NSF-funded project ocused on developing and piloting researchbased formative assessment tools and routines for additive reasoning in grades $\mathrm{K}-3$. In collaboration with the Ongoing Assessment Project (OGAP), we are translating findings from research on student learning of number and operations into tools that are useable for teachers to enhance mathematics instruction. The project includes multiple iterations of design, field-testing, and revision. The materials and routines (including the progressions, a bank of formative assessment items, and professional development materials, have been piloted and over the last 3 years.

Year 1 (2016-17): Materials development \& piloting Year 2 (2017-18): Large-scale field testing

Training provided in Alabama, Vermont, Pennsylvania, Maryland, South Carolina,
Nebraska
Data collection on implementation in Philadelphia hrough interviews, observations, surveys, and a measure of teacher knowledge.

Visual models provide an instructional bridge for students to move from concrete strategies to more abstract addition and subtraction strategies.


ISUAL IIODELS play a key role in bridging students understanding of number as a collection of ones to an understanding of "ten-ness," multi-unit concepts, and the development of more sophisticated addition and subtraction strategies.


NSTRUCTIONAL STRATEGIES Building on work from Dutch researchers (Klein, Beshuizen, \& Treffers 1998) we utilize a ten-structured concrete bead number line to have children locate numbers and develop more sophisticated counting strategies and then transfer that understanding to a number path, number line, and open number line.


Knderigartien studemis use a stick of ten beads to builid an undedistatanding of tete as ten ones and anchor numbers to 5 and 10

FORMATIVE ASSESSMENT Teachers collect evidence of student thinking from formative assessments and analyze student work in relation to the learning progression to develop informed instructional responses. inderstanding of counting, beginning with concrete models and moving towards increasingly abstract addition and subtraction strategies


An inetificient strategy of
ounting on by ones leads the incorrect answer get more information

Eventually, students can
use an efficient adding up strategy without a visual

TEACHER LEARNING
Through professional development we provide eachers with explicit training in the use of these visual models, as well as the role of visual mode in the development of stucent understanding. into their instruction, recognize the models and the purpose in their curricular materials, and use formative assessment items that ask children to interact with visual models to continually assess and respond to students' developing understanding.


ADDITIONAL RESULTS
K-2 teachers need more opportunities to learn about the content they teach and how young children learn mathematics.
Teachers from schools that participated in OGAP training had significantly higher scores than teachers from control schools on the TASK assessment, which measures teacher's ability to analyze and respond to student thinking. (Effect size . $40, \mathrm{p}<.01$ )
Understanding of the OGAP tools and routines varies and is influenced by teachers' beliefs about students, their underlying view of learning, ongoing learning opportunities and principal and school-based support.

## IMPLICATIONS

Explicitly developing this understanding of number lines is an important strategy for number ines is an important strategy for fluency built on conceptual understanding for all children (Ebby, Hulbert, \& Fletcher, 2019).


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