



## **Developing Understanding of the Number Line in Grades K-2**



## INTRODUCTION

Number lines can be a powerful tool to help children make sense of addition and subtraction and develop number sense, understanding of magnitude, and flexible and efficient strategies. While number lines are used in many K-2 math program materials, explicit attention is rarely given to developing understanding of the attributes of number lines so that students can use this model with understanding (e.g., understanding that quantity is represented as a length or distance rather than discrete quantities).

Where does 19 go on the number line?



This student work shows understanding of 19 as the result of counting on from 5, but not in terms of magnitude in relation to the quantity of 5.

## **METHODS**

We are in the third year of a NSF-funded project focused on developing and piloting researchbased formative assessment tools and routines for additive reasoning in grades K-3. In collaboration with the Ongoing Assessment Project (OGAP), we are translating findings from research on student learning of number and operations into tools that are useable for teachers to enhance mathematics instruction. The project includes multiple iterations of design, field-testing, and revision. The materials and routines (including the progressions, a bank of formative assessment items, and professional development materials) have been piloted and implemented in Philadelphia elementary schools over the last 3 years.

Year 1 (2016-17): Materials development & piloting

Year 2 (2017-18): Large-scale field testing

- Training provided in Alabama, Vermont, Pennsylvania, Maryland, South Carolina, Nebraska
- Data collection on implementation in Philadelphia through interviews, observations, surveys, and a measure of teacher knowledge.

Year 3 (2018-19): Data analysis and refinement

## Visual models provide an instructional bridge for students to move from concrete strategies to more abstract addition and subtraction strategies.



The **LEARNING PROGRESSION** builds from children's understanding of counting, beginning with concrete models and moving towards increasingly abstract addition and subtraction strategies.





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A partnership between the University of Pennsylvania Graduate School of Education and OGAPMath LLC

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VISUAL MODELS play a key role in bridging students understanding of number as a collection of ones to an understanding of "ten-ness," multi-unit concepts, and the development of more sophisticated addition and subtraction strategies.







anchor numbers to 5 and 10

FORMATIVE ASSESSMENT Teachers collect evidence of student thinking from formative assessments and analyze student work in relation to the learning progression to develop informed instructional responses.



Take a picture to get more information

(3 + 19 = 81)	d now she has 81 stickers. Iday?
	9 stickers
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An inefficient strategy of counting on by ones leads to the incorrect answer

Natasha had 63 stickers. She got some more stickers for her birthday, and now she has 81 stickers. How many stickers did Natasha get for her birthday? Show how you know.
7+1+10=10 63+18=81
+7 + 10 +1 + 10 $+1$ +1 + 10 $80$ 81
NataSha got Bstickers for her birthday.

The number line supports adding up more efficiently using landmark numbers

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## **INSTRUCTIONAL STRATEGIES**

Building on work from Dutch researchers (Klein, Beshuizen, & Treffers 1998) we utilize a ten-structured **concrete bead** number line to have children locate numbers and develop more sophisticated counting strategies and then transfer that understanding to a number path, number line, and open number line.

Kindergarten students use a stick of ten beads to build an understanding of ten as ten ones and



Eventually, students can use an efficient adding up strategy without a visual model

## **TEACHER LEARNING**

Through professional development we provide teachers with explicit training in the use of these visual models, as well as the role of visual models in the development of student understanding. Teachers learn how to incorporate visual models into their instruction, recognize the models and their purpose in their curricular materials, and use formative assessment items that ask children to interact with visual models to continually assess and respond to students' developing understanding.



## **ADDITIONAL RESULTS**

- K-2 teachers need more opportunities to learn about the content they teach and how young children learn mathematics.
- Teachers from schools that participated in OGAP training had significantly higher scores than teachers from control schools on the TASK assessment, which measures teacher's ability to analyze and respond to student thinking. (Effect size .40, p <.01)
- Understanding of the OGAP tools and routines varies and is influenced by teachers' beliefs about students, their underlying view of learning, ongoing learning opportunities and principal and school-based support.

### **IMPLICATIONS**

Explicitly developing this understanding of number lines is an important strategy for providing equity and access to procedural fluency built on conceptual understanding for all children (Ebby, Hulbert, & Fletcher, 2019).

THE SCHOOL DISTRICT OF PHILADELPHIA



