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Refining a Vision of Ambitious Mathematics Instruction to Address Issues of Equity

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Abstract

The NCTM *Standards* presents an ambitious vision for high-quality mathematics instruction and proposes forms of instructional practice that support the development of students' understanding of central mathematical ideas. We argue that this vision does not provide detailed guidance about how to ensure that instruction is equitable. Equity, in this context, means that all students can participate substantially in all phases of mathematics lessons. The goal of this article is to refine the vision proposed by NCTM to suggest how it could be *equitable* as well as *ambitious*. To this end, we identify several concrete instructional practices likely to support all students' substantial participation in various phases of lessons. Given the limited research base on equitable mathematics instruction, the resulting vision is necessarily provisional and requires further research. We outline a research agenda for identifying additional instructional practices that might support all students to participate substantially in all phases of mathematics lessons.

Refining a Vision of Ambitious Mathematics Instruction to Address Issues of Equity

During the last several decades, the mathematics education research community has developed a robust knowledge base about forms of instructional practice that support the development of students' understanding of central mathematical ideas (Hiebert & Grouws, 2007; Kilpatrick, Martin, & Schifter, 2003; Stein, Remillard, & Smith, 2007). The National Council of Teachers of Mathematics' (2000) *Principles and Standards for School Mathematics* and the more recent *Curriculum Focal Points* (NCTM, 2006) describe a relatively concrete set of learning goals that encompass both conceptual understanding and procedural fluency. In addition, the *Standards* present a vision of mathematics instruction intended to support students' attainment of these learning goals. A number of elementary-, middle-, and high-school mathematics curricula that have been developed with support from the National Science Foundation build on and further specify the instructional goals and forms of classroom practice detailed in the *Standards* (Senk & Thompson, 2003).

The instructional vision proposed in the *Standards* has been called an ambitious vision for high-quality mathematics instruction (Kazemi, 2008; Lampert, Beasley, Ghouseini, Kazemi, & Franke, 2010). This article is premised on the claim that, despite the best intentions of their developers, the *Standards* do not provide detailed guidance about how to ensure that instruction is also *equitable* (cf. Boaler, 2002). Equity, in this context, means that all students can participate substantially in all phases of mathematics lessons (e.g., individual work, small group work, whole class discussion), but not necessarily in the same ways.

In what follows, we refine the vision of high-quality mathematics instruction presented in the *Standards* with the goal of ensuring that it *equitable* as well as *ambitious*. We do so by building on the limited research available on equitable mathematics instruction as well as our current work (which we describe below). As will become clear, the resulting vision of ambitious and equitable mathematics instruction is necessarily provisional and requires further research to fill it out. In the latter part of this article, we therefore detail a research agenda for identifying additional concrete instructional practices that might support all students to participate substantially in all phases of lessons.

Ambitious Mathematics Instruction

The current vision of high-quality mathematics instruction presented in the NCTM (2000) *Standards* proposes several content- and process-oriented goals for student learning (see also Kilpatrick, Swafford, & Findell, 2001). For example, students should develop conceptual understanding of key mathematical ideas and procedural fluency in a range of domains (e.g., number and operations, algebra, geometry, measurement, data analysis and probability). Additionally, students should master increasingly sophisticated forms of mathematics argumentation (including methods of proof) and should learn to communicate their mathematical reasoning effectively by using multiple representations (e.g., words, graphs, tables) and by making connections between different representations.

Research on mathematics teaching has identified a coherent set of instructional practices for supporting students' achievement of these goals (Franke, Kazemi, & Battey, 2007; Hiebert, et al., 1997). The instructional vision represented in the *Standards*

includes recommendations about the nature of tasks and about classroom norms and discourse. For example, the *Standards* recommend that teachers pose cognitively demanding tasks that require students to explain their reasoning (Stein, Smith, Henningsen, & Silver, 2000). In addition, the *Standards* emphasize the value of tasks that have “multiple entry points,” meaning that they “can be approached in more than one way, such as using an arithmetic counting approach, drawing a geometric diagram and enumerating possibilities, or using algebraic equations, which makes the tasks accessible to students with varied prior knowledge and experience” (NCTM, 2000, p. 18). The *Standards* recommend that the teacher implement such tasks in the classroom by first giving students time to work individually and/or in groups, and then leading a whole class discussion in which students explain and justify their solutions. During the discussion, the teacher should select and sequence the types of solutions that are shared to ensure that conversation focuses on key mathematical issues that advance the instructional agenda (Stein, Engle, Smith, & Hughes, 2008). In addition, the teacher should press students to make connections between different solutions, and should mediate the communication between students to help them understand each others’ explanations (McClain, 2002).

This vision of instruction implies that a particular set of social norms and sociomathematical norms should be established in the classroom (Yackel & Cobb, 1996). Although classroom norms are jointly constituted by the teacher and students, it is the teacher’s responsibility to initiate and guide the establishment of norms that support his or her agenda for student learning (Cobb, Stephan, McClain, & Gravemeijer, 2001). Social norms that are potentially productive for students’ learning include that students

should explain how they solved tasks, connect their solutions and reasoning to others' solutions, and indicate points of agreement and disagreement with others' ideas (Franke, et al., 2007). Potentially productive sociomathematical norms include that students should explain not merely how they solved a task but why they used particular methods and how a given solution differs from others' solutions (Franke, et al., 2007).

Equity in Student Learning Opportunities

There is broad consensus within the mathematics education research community on the vision of high-quality mathematics instruction presented in the *Standards*. This vision acknowledges that the current distribution of classroom learning opportunities is inequitable, particularly for students living in poverty, students of color, students for whom English is not their first language, and students who have been identified as in need of Special Education services (Cobb & Nasir, 2002; Gutstein, et al., 2005; Tate, 1994). The *Standards* include an “equity principle” that is intended to orient the development of more equitable instructional practices:

All students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study—and support to learn—mathematics. Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students. (NCTM, 2000, p.12)

However, the guidance provided in the *Standards* (2000) is limited to broad suggestions about “reasonable and appropriate accommodations” that might support all students to participate substantially in mathematical activity in the classroom. The suggestions

include the following: communication of high expectations in the classroom and through contact with students' caregivers, "access to an excellent and equitable mathematics program that provides solid support for [students'] learning and is responsive to their prior knowledge, intellectual strengths, and personal interests" (p. 12), provision of supplemental learning opportunities (e.g., through tutoring), and use of technology. For English language learners (ELLs) in particular, the *Standards* recommend both "special attention to allow them to participate fully in classroom discussions" and assessment accommodations (p. 12). Additionally, the *Standards* observe that teachers "have to confront their own biases" about students from diverse backgrounds (p. 12).

In our view, it is understandable that the *Standards* do not propose concrete instructional practices that might support all students' substantial participation in classroom mathematical activities. At the time the *Standards* were written, research on concrete instructional practices specific to supporting all students' participation was limited (Boaler, 2002). In the absence of such research, the position taken in the *Standards* is, in effect, that "good instruction for one is good instruction for all." In other words, it is assumed that all students will have sufficient opportunities to learn if instruction is generally compatible with in the vision of ambitious instruction presented in the *Standards*. We contend that assumption is flawed.

Boaler (2002) and others (e.g., Gresalfi, Taylor, Hand, & Greeno, 2008; Nasir & Cobb, 2007) clarify that any vision of instruction, including that proposed in the *Standards*, places value on particular ways of engaging in mathematical activity. For example, the vision of high-quality instruction presented in the *Standards* places value on specific types of verbal communication (e.g., explaining the process of arriving at an

answer). These forms of engagement necessarily involve culturally specific assumptions about what is normal, reasonable, and self-evident. The extent to which these assumptions are transparent to students will vary depending on their instructional histories and on the types of activities in which they have participated outside the classroom (Nasir, Hand, & Taylor, 2008). Some students but not others will therefore need explicit support if they are to learn the “rules of the game” and thus be effective in the mathematics classroom (Boaler, 2002). Framed in the way, the task of refining the vision of ambitious instruction proposed in the *Standards* to address issues of equity involves specifying accommodations that teachers might make to enable all students to participate substantially in classroom mathematical activities.

A Provisional Vision of Ambitious and Equitable Mathematics Instruction

Our goal in proposing a provisional vision of ambitious and equitable instruction is to specify *concrete* forms of instructional practice that, we conjecture, are learnable in the context of high-quality teacher professional development. We hope that in addition to providing an orientation for classroom practice, the vision that we outline might also provide an orientation for teacher professional development and for efforts to improve the quality of instruction more generally. We take a vision of ambitious instruction, as presented in the *Standards*, as our starting point, and we ask the following questions:

1. What aspects of participation in the envisioned types of classroom activities are likely to be difficult for students who have not previously engaged substantially in such activities?

2. How might the envisioned types of classroom activities be modified and what additional supports might be provided to enable all students to participate substantially?

We developed the provisional vision of ambitious and equitable mathematics instruction by conducting an exhaustive review of relevant studies in mathematics education, educational psychology, and the sociology and anthropology of education. In addition, we drew on an analysis of video-recordings of middle-grades mathematics classrooms in which teachers used a *Standards*-based curriculum and in which there was atypical growth in student achievement for traditionally low-performing groups of students.¹ We used two criteria when deciding whether to incorporate particular instructional practices into this provisional vision of ambitious and equitable instruction. First, it was necessary that the forms of practice be relatively concrete and that they have been shown to support traditionally low-performing groups of students' substantial participation in classroom activities likely to result in their attainment of mathematical learning goals proposed in the NCTM's (2000) *Standards*. We therefore looked carefully at the evidence of supporting students' substantial participation provided in research reports. Unless otherwise noted, any study that we cite described qualitative changes in students' opportunities to learn mathematics and/or students' participation in

¹ We are currently involved in a research project that seeks to understand what it takes to support instructional improvement on a large scale. To this end, we are attempting to identify district- and school-based organizational arrangements, material resources, and social relations that support middle-grades mathematics teachers' development of ambitious forms of instructional practice (see Cobb & Smith, 2008). We are collaborating with four, large urban districts that have set ambitious instructional goals for student learning in the middle grades and have developed relatively coherent plans for supporting teachers' development of compatible instructional practices. The data that we are collecting during each of the four years of the study include video-recordings of two consecutive lessons taught by each of the 120 participating mathematics teachers (30 teachers in each of 4 districts). We completed the literature review and analyzed the video-recordings in order to develop an instrument that will enable us to assess the extent to which instruction supports traditionally low-performing groups of students' participation in rigorous mathematical activity.

mathematical activity. Additionally, we noted when research reported overall increases in student performance and a reduction in gaps between traditionally low-performing and high-performing groups of students. Second, it was necessary that the forms of instructional practice were potentially learnable by most mathematics teachers, meaning that we could imagine how professional development could be designed to support teachers' development of the practices. Although a number of relevant studies have been reported in the decade since the *Standards* were published, the research base on concrete instructional practices that support all students' participation is still limited. As a consequence, the vision of ambitious and equitable instruction that we propose is necessarily incomplete.

We organize our presentation of the provisional vision in terms of the typical phases of a *Standards*-based mathematics lesson (Stein, et al., 2008): the teacher poses a cognitively-demanding task, the students work to solve the task either individually or in groups, and the teacher leads a whole class discussion of the students' solutions.

Posing the Task

As we have noted, mathematics education reform proposals emphasize cognitively demanding tasks as a primary means of supporting students' development of relatively sophisticated understandings of key mathematical ideas (Boston & Smith, 2009; Stein, et al., 2000). The tasks included in *Standards*-based curricula frequently involve real world contexts and other types of scenarios that are intended to support both students' initial engagement and their subsequent learning. In order for the designers' intentions to be realized in the classroom, it is essential that students experience the task scenario as real so that they can "evoke the imagery of situations described in problem

statements when solving tasks” (McClain & Cobb, 1998, p. 60). However, as Boaler (2002) observed, “one of the problems presented by real-world contexts is that they often require familiarity with the situation that is described, but such familiarity cannot always be assumed” (p. 251). From an equity perspective, a lack of familiarity with the suppositions of the task scenario is likely to limit students’ participation when working on solving the problem (Ball, Goffney, & Bass, 2005; Boaler, 2002; Lubienski, 2000; Silver, Smith, & Nelson, 1995).

Lessons consistent with the vision of ambitious instruction proposed in the *Standards* typically begin with the teacher posing, or launching, the main task for the day. Based on a study of teachers who were successful in supporting the participation of traditionally low-performing students, Boaler (2002) argued that the teacher should lead a whole class discussion to introduce the task and, in doing so, should “decide on the degree of support or structure the students need” to begin to solve the task (p. 248).

Building on Boaler’s (2002) findings, and based on our analysis of classroom video-recordings, we distinguish between two aspects of task-posing that we conjecture are likely to support all students in engaging productively in the task. First, in introducing the task, the teacher should support all students’ understanding of the cultural suppositions inherent in the task scenario (cf. Ladson-Billings, 1995). Second, the teacher should support students’ development of situation-specific imagery of the mathematical relationships described in the task statement. Our attention to students’ mathematical images is based on the work of Thompson (1996) and McClain and Cobb (1998), who argued that students’ initial understanding of the mathematical relationships described in the task statement provide a basis for any mathematizations that they might make as they

attempt to solve the task. Summarizing Thompson's (1996) findings, McClain and Cobb (1998) concluded that in the absence of situation-specific mathematical imagery, students' efforts to solve tasks typically become "decoupled" from their interpretations of problem situations" (p. 65).

We ground our discussion of these two aspects of task posing by focusing on one of the video-recorded lessons that we analyzed. The majority of the seventh-grade students in the illustrative case received free or reduced-price lunches. Most were Latino/a, several of whom were designated as ELLs; the class also included African American and White students. As a group, the students' value-added scores on the state mathematics assessment indicated better than expected growth. The teacher posed the task shown in Figure 1 about midway in the school year. Earlier in the year, the students had solved problems that involved linear relationships by creating and using tables, graphs, and equations. The novel feature of this lesson was that the y-intercept for all of the linear relationships was not at 0.

Dollars for Dancing

Three students at a school are raising dollars for the school's Valentines Dance. All three decide to raise their money by having a dance marathon in the cafeteria the week before the real dance. They will collect pledges for the number of hours that they dance, and then they will give the money to the student council to get a good DJ for the Valentines Dance.

Rosalba's plan is to ask teachers to pledge \$3 per hour that she dances.

Nathan's plan is to ask teachers to give \$5 plus \$1 for every hour he dances.

James's plan is to ask teachers to give \$8 plus \$0.50 for every hour he dances.

Part A. Create at least three different ways to show how to compare the amounts of money that the students can earn from their plans if they each get one teacher to pledge.

Part B. Explain how the hourly pledge amount is represented in each of your ways from Part A.

Part C. For each of your ways in Part A explain how the fixed amount in Nathan’s plan and in James’s plans is represented.

Part D. For each of the ways in Part A show how you could find the amount of money collected by each student if they could dance for 24 hours.

Part E. Who has the best plan? Justify your answer.

Figure 1. Mathematical Task, “Dollars for Dancing.”²

Key suppositions of the task scenario include knowing what a dance marathon is as a social event, why people organize and participate in dance marathons, and the role pledges play in dance marathons. Key situation-specific mathematical images include that money accumulates as a participant continues to dance for a greater number of hours (A. G. Thompson & Thompson, 1996). Our extensive viewing of instances of teachers posing tasks involving real world scenarios indicates that it is common for teachers to attend to the suppositions inherent in the scenario (although far more superficially than in our illustrative case). In contrast, it is much less common for teachers to support students’ development of situation-specific imagery of the mathematical relationships described in the task statement. However, we contend that it is crucial for teachers to attend to both aspects. In the case of the illustrative task, understanding how money accumulates over time is as important as knowing what a dance marathon is and what it means to make a pledge. Without both understandings, it is probable that some students will not be able to substantially participate in solving the task.

Returning to the illustrative case, one aspect of the teacher’s expertise concerned the carefully planned way in which he introduced tasks. For example, he made suppositions inherent in the task scenario transparent prior to introducing the task

² The teacher adapted this task from Task 1.2 (“Raising Money”) in the *Connected Mathematics Project 2* grade 7 book, *Moving Straight Ahead: Linear Relationships* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2009).

statement by projecting several images of dance marathons and asking students to tell him what they knew about the images. He then capitalized on students' contributions to develop a provisional description of dance marathons as "groups of people who dance for a certain amount of time." The teacher also pressed the students to explain why people might hold a dance marathon and then built on several of their proposals to explain that the task they would solve today involved holding a dance marathon to raise money.

Additionally, the teacher explicitly supported the students' development of situation-specific imagery of key mathematical relationships described in the task statement. He first described to the students two ways of raising money:

T³: We, when we talk about dance, and dance marathon and raising money there's two ways you can raise money in a dance marathon that we're going to talk about. One way is to dance for a long time. M/St1 and some others said a marathon takes a long time and people dance for a long time. So if you dance for a long time, and let's say I give you 50 cents every hour you're going to make a lot of money. But there's another way that you could raise money and that is to ask for a pledge. Not per hour, but just a donation. Okay we call that a donation. And you might go up to your teacher and say, can you give me \$6 for being in the dance marathon. Now that's different. Can anybody explain why how that is different if I say can you give me \$6 or instead can you give me 50 cents an hour?

As the exchange continued, the teacher guided the explicit negotiation of relationships between the quantities described in the task statement. In doing so, he asked students to

³ Within the transcript, T refers to the teacher, and M/St and F/St refer to male and female students, respectively. Different students are distinguished by the use of numbers depending on the order in which they spoke in the lesson (e.g., M/St1 is the first male speaker; M/St10 is the tenth male speaker).

restate what he and others had said, and took up students' ways of describing relationships.

T: What's the difference M/St10?

M/St10: Because you start with some money and then they add more money...

T: F/St4, add on to that.

F/St4: It's like, either they pay you up front or you continue so like they continue to pay you for however long you dance.

T: Great. So we have one where they pay you up front, one where they add on to it. How many people understand kinda what this is...what we're talking about here? [waits to see students' hands]. Two kinds of fund raising. Pay you up front or pay you where you add on. F/St6, can you say it in your words? There's two ways that you could raise money, what are they?

F/St6: Well like one of them you already start with it and the other one you have to kind of work for it to get more.

T: Exactly. I like the way that's worded. One of them you start with it, you just have it. The other one you got to work for it to get the money.

We have described and illustrated a concrete instructional practice that is specific to leading a whole class discussion during the task-posing phase of lessons prior to students working on the task. The intent of the discussion is to support students in understanding the suppositions implicit in the task scenario and in developing situation-specific images of key mathematical relationships described in the task statement. We argue that attending to both types of understanding is necessary if all students are to participate in solving the task at hand. As an orientation for both practice and

professional development, we therefore suggest that teachers consider the following two questions in order to decide what to focus on when posing a task:

1. What suppositions implicit in the task scenario might be unfamiliar to some of my students?
2. What must students understand mathematically about the task in order to begin attempting to solve it productively?

A second, related issue concerns the importance of the teacher *explicitly* negotiating with students the norms of participation for all phases of a lesson including the initial problem-posing phase. Research on equitable teaching practices in general, and in mathematics classrooms in particular, indicates that the classroom norms inherent in ambitious instruction might be novel to certain groups of students (Boaler, 2002; Boaler & Staples, 2008; Delpit, 1995; Ladson-Billings, 1995; Murrell, 1994). The teacher can address this potential source of inequities in learning opportunities by supporting all students in coming to understand their obligations to the teacher and each other, and in coming to view these obligations as reasonable. In the case of the illustrative seventh-grade classroom, the posing of the task proceeded smoothly and it was evident that all the students realized that they were obliged to listen carefully to others' contributions and to re-state others' ideas in their own words. Early in the school year, the teacher of the illustrative lesson might well have discussed and developed a rationale for these obligations with his students. We will return to the process of explicitly negotiating norms of participation when we discuss the remaining two phases of the lesson.

Students Solving the Task

During the second phase of a lesson, students might work individually, in pairs, or in groups of three or four to solve the task. Most if not all of the research particular to this phase and relevant to issues of equity focuses on the social organization of cooperative groups. It is worth stating explicitly that the instructional practices that we discuss for supporting all students' substantial participation in cooperative groups should be understood against a particular set of goals for having students work in groups. Together with Wood and Yackel (1990), we frame the rationale for cooperative grouping in terms of the "meaningful negotiation of mathematical viewpoints and solutions" (p. 245). Our focus is therefore on the extent to which students' participation in small group activities supports their development of conceptual understanding and procedural fluency. We first describe research that indicates characteristics of small group interactions that provide equitable learning opportunities for students. We then describe concrete instructional practices that are likely to support the development of equitable small group interactions.

Cobb (1995) details characteristics of small group activities that provide equitable opportunities for students to develop conceptual understanding and procedural fluency. Based on an analysis of a second-grade classroom that focused on four pairs of students' small group work over a ten-week period, Cobb describes the relationship between the nature of students' small-group interactions and the extent to which learning opportunities arose for the students. The aspect of the analysis that is most relevant to our purposes focused on interactions that occurred after one or both students had arrived at a solution and "one child attempt[s] to explain his or her thinking to the other, or the children [attempt] to resolve conflicts among their interpretations, solutions, and answers" (p. 42).

In these instances, Cobb found that *multivocal* interactions in which no child is established as either a *mathematical authority* or *social authority* frequently gave rise to learning opportunities. In contrast, Cobb reported that univocal interactions in which “the perspective of one child dominates” (p. 42) rarely gave rise to learning opportunities for either child. As he clarified, one child explaining a solution while the other listens did not usually further either child’s mathematical understandings. (See Esmonde, 2009b for a similar account of what she refers to as “helping interactions.”) Univocal interactions typified small group relationships in which one child had been constituted as the *mathematical authority* in the group and “there was a clear power imbalance between the children in that one child was obliged to adapt to the other’s mathematical activity in order to be effective in the group” (Cobb, 1995, p. 43). The only instances that Cobb identified in which univocal interactions were productive occurred when the dominant child “clarifies and organizes his or her [own] thinking while explaining a solution in new and different ways” (p. 106).

Following Wertsch (1990), Cobb characterized multivocal interactions as those in which “conflict has become apparent and both children insist that their own reasoning is valid....[B]oth children attempt to advance their perspectives by explicating their own thinking and challenging that of the partner” (Cobb, 1995, p. 42). Cobb found that multivocal interactions frequently gave rise to learning opportunities provided the students had established a basis for mathematical communication and no one child was established as a social authority in the group. *Social authority* refers to cases in which one child “regulates the way in which the children interact as they do and talk about mathematics” (p. 43). Evidence that a social authority has been established is when

“children engage in multivocal interactions only when a discussion of conflicting situations fits with one child’s personal agenda” (p. 43). Cobb clarified that the establishment of a social authority in a group results in inequities in learning opportunities.

We now turn to research that specifies concrete instructional practices that might support multivocal interactions in which no child is established as a mathematical or social authority in the context of small group activity. As Esmonde (2009a) indicates, research that specifies concrete instructional practices that support all students’ participation in productive cooperative work is limited. Nonetheless, several studies provide useful guidance, including Wood and Yackel’s (1990) analysis of the teacher’s role during small-group work in the same second-grade classroom studied by Cobb (1995).

Wood and Yackel (1990) described how the teacher explicitly scaffolded pairs of students in collaborating productively to solve high-cognitive demand tasks. In doing so, the teacher supported the development of the following norms for small group work: students were obliged to verbalize solutions to one another, students were obliged to listen to alternative solutions that the partner offered, and students were obliged to reach consensus about solutions. Wood and Yackel (1990) illustrated how the teacher intervened to negotiate norms for small-group work explicitly with students. Their analysis highlighted the teacher’s role in listening to students’ exchanges and interjecting to *maintain the dialogue* between students. Crucially, they clarified that the intent of the teacher’s interjection was *not* be to explain one student’s solution to another student. Instead, their analysis implies that teachers should make comments or ask questions in

order to support the students in verbalizing their solutions, listening to others' solutions, and reaching consensus about solutions. The instructional practices that Wood and Yackel (1990) documented are important in fostering the students' development of a basis for mathematical communication. These practices are also important from an equity perspective because they indicate concrete ways in which a teacher can explicitly negotiate with students how to participate effectively in this phase of a lesson.

Boaler and Staples' (2008)⁴ analysis of high school classrooms in which small-group work reflected the principles of Complex Instruction is relevant in specifying ambitious and equitable instructional practices. Complex Instruction is not specific to mathematics and was developed to support students in working together productively in heterogeneous groups (Cohen, Lotan, Scarloss, & Arellano, 1999). Boaler and Staples (2008) identified a specific instructional practice specific to the teacher's role, assigning social competence, which fosters multivocal interactions in which no child is established as a mathematical or social authority. Boaler and Staples (2008) clarified that assigning competence is a principle of Complex Instruction and explained that it

involves teachers raising the status of students [who may be perceived by other students as having little to contribute intellectually], by, for example, praising something they have said or done that has intellectual value, and bringing it to the group's attention; asking a student to present an idea; or publicly praising a student's work in a whole class setting. (p. 632)

As Boaler and Staples (2008) illustrated, a teacher might publicly highlight, or mark, a contribution made by a student who is typically quiet or is marginalized in the

⁴ Boaler and Staples (2008) provide evidence that students' performance on mathematical assessments improved over time, and that disparities in achievement between ethnic and racial groups of students diminished over time.

group. They clarified that if the practice of assigning competence is to address status issues,

it must be public, intellectual, specific, and relevant to the group task (Cohen, 1994, p. 132). The public dimension is important as other students learn about the broad dimensions that are valued; the intellectual dimension ensures that the feedback is an aspect of mathematical work; and the specific dimension means that students know exactly what the teacher is praising. (Boaler & Staples, 2008, p. 633)

The number of studies that have addressed the issue of how the small group participation of students whose first language is not English can be supported is extremely limited. Gutiérrez (2002) analyzed small group work in high school calculus classrooms in a school in which a significant number of Latino/a students took advanced mathematics courses. Although the students were primarily English-dominant, many were proficient in Spanish. She found that the establishment of the norm of using either Spanish or English in small group work contributed to the students' effectiveness in communicating mathematically in the classrooms she observed. This finding is consistent with the conclusions of Franke, Kazemi, and Battey's (2007) review of studies that have focused on supporting ELLs' participation in rigorous mathematical activity. They suggested that it is important to allow ELLs to use their first language in small group work but also pointed to the teacher's role in scaffolding students' development of English mathematical language. In doing so, they drew on Moschkovich's (2002, as cited in Franke, Kazemi & Battey, 2007) work to highlight the instructional practice of linking

the mathematical ideas students express in their home language with English mathematical language.⁵

Whole Class Discussion

The final phase of lessons that aim at ambitious instructional goals typically involves a whole class discussion of students' solutions. As Stein et al. (2008) clarify, a productive discussion is one in which the teacher "effectively guide[s] whole-class discussions of student-generated work toward important and worthwhile disciplinary ideas" (p. 319). It is important to note that the quality of the two preceding phases of a lesson influence the extent to which the whole class discussion can be productive. For example, if the teacher poses the task effectively, students can attempt to solve the task with little if any support. This in turn enables the teacher to plan for the whole class discussion during individual or small group work by focusing on the different ways in which students are attempting to solve the task. Furthermore, students' expectations about what the teacher will hold them accountable for during whole class discussions influence how they participate in small group work (Cobb & Bauersfeld, 1995). It is therefore crucial that the teacher communicates clear expectations about what students will be held accountable for individually and collectively both while working in groups and during subsequent phases of the lesson (e.g., all students should be able to explain the groups' work) (Boaler & Staples, 2008).

Research on ambitious mathematics instruction has identified several types of social and sociomathematical norms that, if established, are likely to improve the quality

⁵ Moschkovich (1999, 2002) did not describe qualitative changes in opportunities to learn or students' participation in classrooms as a result of the teacher's scaffolding of ELLs' language development. We include her work in this review because she illustrates specific instructional practices that support ELLs' participation in small group and whole class discussion.

of the learning opportunities that arise for students during whole class discussions. These norms include that students are obliged to explain and justify their solutions, explain why they used one particular method rather than another, and articulate the relationship between different solution strategies (Cobb, et al., 2001; Silver & Stein, 1996). In addition, Stein et al. (2008) described five specific instructional practices that are integral to orchestrating whole class discussions that support students' understanding of significant mathematical ideas:

- 1) anticipating likely student solutions to cognitively demanding mathematical tasks;
- 2) monitoring students' responses to the tasks during the explore phase;
- 3) selecting particular students to present their mathematical responses during the discuss-and-summarize phases;
- 4) purposefully sequencing the student responses that will be displayed; and
- 5) helping the class make mathematical connections between different students' responses and between students' responses and key ideas. (p. 321)

In discussing how teachers might refine their classroom practices to support all students' substantial participation in whole class discussions, we first note that it is critical that the teacher explicitly negotiate with students how to participate in this phase of lessons (Boaler, 2002; Murrell, 1994). A concern for all students' participation also has implications for the teacher's decisions about which students to call upon to share their solutions. Suggestions for additional refinements of the teacher's role focus on supporting students' access to each other's mathematical reasoning. In speaking of access, we do not mean that students are merely given an opportunity to hear what others are thinking. Instead, we mean that a whole class discussion provides all students,

including students who are currently struggling with the particular mathematical ideas at hand, with adequate supports so that they might understand others' explanations. In this regard, the norms or standards for what counts as an acceptable mathematical explanation that are established in the classroom appear to be crucial.

Building from the work of A.G. Thompson, Phillip, Thompson and Boyd (1994), Cobb and his colleagues (2001) described a distinction between two types of classroom discourse that they referred to as calculational discourse and conceptual discourse. They emphasize that calculational discourse is not restricted to conversations that focus merely on the procedural manipulation of conventional mathematical inscriptions. Instead, calculational discourse refers to discussions in which the primary topic of conversation is any type of process that is enacted to arrive at a result. These conversations can be contrasted with conceptual discourse in which the reasons for carrying out solution processes also become an explicit topic of conversation. In this latter case, conversations encompass both the process of producing results and the underlying task interpretations that motivate those processes and that constitute their rationale. Analyses of elementary and middle school classrooms in which students were obliged to articulate their task interpretations indicate that conceptual discussions can be productive settings for mathematical learning (Cobb, McClain, & Gravemeijer, 2003; Cobb, et al., 2001). When compared with calculational explanations, conceptual explanations increase the likelihood that listening students might come to understand the explainer's thinking because they include an explicit account of the task interpretations that underpin particular solution strategies. As a consequence, conceptual explanations provide

additional supports for students who interpreted the task in a different or less sophisticated manner when they attempted to solve it.

The development of conceptual discourse in the classroom involves renegotiating the sociomathematical norm of what counts as an acceptable mathematical explanation. McClain and Cobb (1998; see also A.G. Thompson, et al., 1994) reported the case of a teacher who obliged her students to articulate their task interpretations by consistently pressing them to ground their explanations in the situation-specific images of the key mathematical relationships (A. G. Thompson, R. A. Philipp, P. W. Thompson, & B. A. Boyd, 1994). Pressing students to ground their explanations in situation-specific images proved to be critical in instances where it had become apparent that some of the students were having difficulty in understanding other's explanations. McClain and Cobb (1998) also described how there was gradual hand over of responsibility in the classroom such that the students began to press each other to explain not merely how they had solved the task, but why they had solved it in a particular way.

We conjecture that conceptual discourse will support students' access to one another's reasoning in small group exchanges as well as in whole class discussions. If this is the case, it will be important for the teacher to press students to ground their explanations in situation-specific images of mathematical relationships that they developed when the task was posed during these phases of the lesson.

The issue of how to support all students' access to other's mathematical reasoning is clearly related to that of supporting ELLs' acquisition of English in the context of doing mathematics (Secada, 1996). Our discussion of instructional practices that support language acquisition focuses on students' participation in whole class discussions.

However, because issues of language acquisition impact the extent to which ELLs can participate in all phases of the lesson, the practices we describe also apply to small-group work and to the initial task-posing phase.

Moschkovich (1999) has proposed that in cases where students use informal or non-mathematical language to explain their reasoning, the teacher can rephrase or revoice their explanations “in ways that are closer to the standard discourse practices of the discipline” (p. 15). Moschkovich argued that this type of support is particularly important for ELLs because revoicing can serve to bridge between students’ informal, everyday language and more formal mathematical language. Additionally, revoicing can potentially mark a student’s contribution as legitimate (Moschkovich, 1999; see also Franke, et al., 2007), thereby serving as a means of assigning competence to particular students.

Conclusion

We have argued (as have others, see Boaler, 2002; Secada, 1996) that the vision of ambitious mathematics instruction reflected in the NCTM (2000) *Standards* does not adequately support all students’ substantial participation in rigorous mathematical activity. We have addressed this limitation by proposing a number of specific instructional practices that elaborate this vision. The instructional practices that we have discussed include: supporting students in understanding the cultural suppositions of the task scenario and in developing situation-specific images of mathematical relationships described in the task statement; guiding students’ development of small group interactions characterized by multivocal interactions; guiding the development of conceptual rather than calculational discourse; initiating and guiding the explicit

negotiation of how to participate in all phases of the lesson; rephrasing or revoicing students' reasoning expressed using informal or non-mathematical language explanations in terms of formal, mathematical language; and supporting students in being recognized as mathematically competent (see Table 1).

Table 1

Equitable Instructional Practices

Typical phases of a Standards-based lesson	Equitable instructional practices
Teacher poses a cognitively-demanding task	Teacher holds a whole class discussion aimed at supporting students' 1) understanding of the cultural suppositions inherent in the task scenario (Boaler, 2002, Ladson-Billings, 1995) and 2) development of situation-specific imagery of the mathematical relationships described in the task statement (Thompson, 1996).
Students work on solving the task either individually or in small groups	Teacher guides students' development of small group interactions that are characterized by multivocal interactions. For example, once students begin to share individual explanations, teacher listens to small group interactions and interjects to maintain the dialogue between students (e.g., ask questions or make comments to support students to verbalize solutions, listen to others' solutions, and reach consensus about solutions) (Yackel & Wood, 1990).
Teacher leads a whole class discussion of the students' solutions	Teacher presses and supports students to engage in calculational discourse (e.g., the reasons for carrying out solution processes also become an explicit topic of conversation, students' explanations are grounded in situation-specific images of the key mathematical relationships. This requires that the teacher re-negotiates with students sociomathematical norm of what counts as an acceptable solution (Cobb et. al, 2001).
<i>Applies to each phase</i>	Teacher explicitly negotiates with students the norms of participation in each phase of the lesson, including what students will be held accountable for in each phase of the lesson (Boaler & Staples, 2008). Teacher assigns competence (e.g., teacher publicly

highlights or marks a contribution made by a student who is typically quiet or marginalized) (Boaler & Staples, 2008).

In cases where students use informal or non-mathematical language to explain their reasoning, the teacher rephrases or revoices their explanation in terms of formal mathematics language (Moschkovich, 1999, 2002).

This elaborated vision of high-quality mathematics instruction is necessarily incomplete because the number of studies that identify concrete instructional practices specific to supporting all students' participation is still limited. In our view, the need for additional studies that investigate how teachers can explicitly negotiate with students how to participate in all phases of the lesson is particularly pressing. To this point, only a few studies have analyzed the negotiation of norms in classrooms in which groups of traditionally low-performing students participated effectively (for an exception, see Boaler & Staples, 2008). Within this limited group of studies, work that focuses on supporting either the development of equitable small group relationships (cf. Esmonde, 2009a) or ELLs' acquisition of English in the context of learning mathematics and their access to other students' reasoning is significantly underrepresented.

As we indicated earlier, one of our criteria for identifying forms of instructional practices is that they are potentially learnable by most mathematics teachers, meaning that we can imagine how professional development could be designed to support teachers' development of the practices in question. Grossman and McDonald (2008) have suggested that teacher education should focus on *pedagogies of enactment* that provide opportunities for pre- and in-service teachers to "rehearse and develop discrete components of complex practice in settings of reduced complexity" (p. 190). Designing,

testing, and refining pedagogies of enactment for particular instructional practices might in fact be a useful way of operationalizing what is potentially learnable by teachers. For example, in the case of the instructional practices that we have discussed related to problem posing, we can imagine designing a laboratory-like setting in which pre- and in-service teachers are supported in analyzing instances of task posing, planning for the posing of particular tasks to specific groups of students, and enacting posing the tasks.

A substantial body of research on mathematics teacher learning indicates that teachers' development of ambitious forms of instructional practices is complex and demanding (Stein, et al., 2000). Teachers' development of instructional practices that are both ambitious and equitable further increases these demands. Teachers will surely need high-quality professional development that is oriented by an ambitious and equitable vision of mathematics instruction if they are to develop the proposed instructional practice. However, professional development will not, by itself, be sufficient because its impact on teachers' classroom practices is mediated by the school and district settings in which teachers work (Coburn, 2003; Spillane, 2005; Stein, 2004). It is therefore essential to take a broader perspective on supports for teachers' learning that includes school and district organizational arrangements (e.g., regularly scheduled time for teacher collaboration), social relationships (e.g., access to colleague(s) who have already developed teaching that are practices ambitious and equitable), and material resources (e.g., supplementary instructional materials) as well as formal professional development. We therefore suggest that in addition to identifying potentially learnable instructional practices and testing and refining professional development designs, future research on

ambitious and equitable mathematics instruction should also seek to identify school and district supports for teachers' ongoing learning.

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