Designing, Refining, and Validating LTs/LPs in Early Algebra

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The research reported here was supported in part by the National Science Foundation under DRK-12 Awards # 1207945 and DRL 1154355. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

Challenge 1: Finding clarity

- Understanding what it is that we need to design (what does it include?)
- What does it *mean* to validate this?

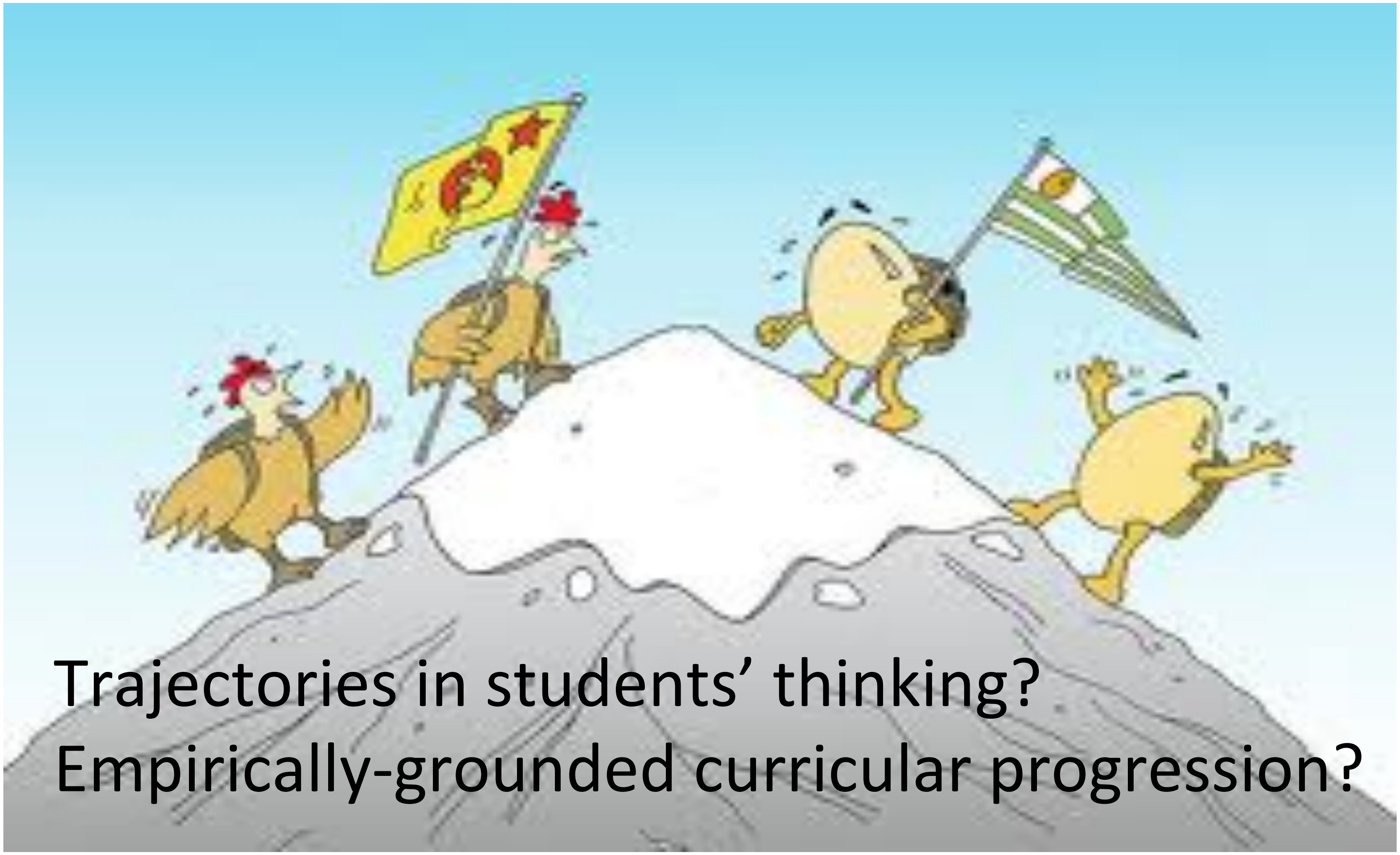
Assumptions about what LPs/LTs include

- successively more sophisticated ways of thinking about a concept over a “broad” span of time
- tasks or instructional sequence that support development
- outcomes (evidence)

(Anderson et al, 2011)

- ….or maybe something else

- *What* are we designing?
- *What* are we validating?



Trajectories in students' thinking?
Empirically-grounded curricular progression?

BOTH, ultimately?

BOTH, ultimately?

Does something get foregrounded?

Learning through an Early Project LEAP Algebra Progression

Project LEAP:

Primary goal is to develop tools from which to measure the impact of early algebra education on children's algebra-readiness.

- Developed an empirically-based "new" instructional sequence <u>across grades 3-7</u> integrating multiple domains of early algebra;
- We "validated" the sequence (curricular progression) by
  - implementing the sequence through year-long classroom teaching experiments in grades 3-5; ongoing analysis re students' understanding and refinement of subsequent lessons
  - retrospective analysis of all lessons to refine instructional sequence
  - qualitative analysis of students' strategies exhibited in written work and classroom observations;
  - quantitative analysis of students' performance on written assessments

Was our "hypothesized" sequence a reasonable sequence?

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  - retrospective analysis of all lessons to refine instructional sequence
  - qualitative analysis of students' strategies exhibited in written work and classroom observations;
  - quantitative analysis of students' performance on written assessments
- We are identifying profiles in students' thinking ("levels of sophistication") as a result of year-long instructional sequences.

Children's Understanding of Functions (CUF):

Primary goal is to develop (1) trajectories that characterize how children's thinking, <u>in each of</u> grades K-2, develops regarding functional relationships and (2) the instructional sequence that supports this.

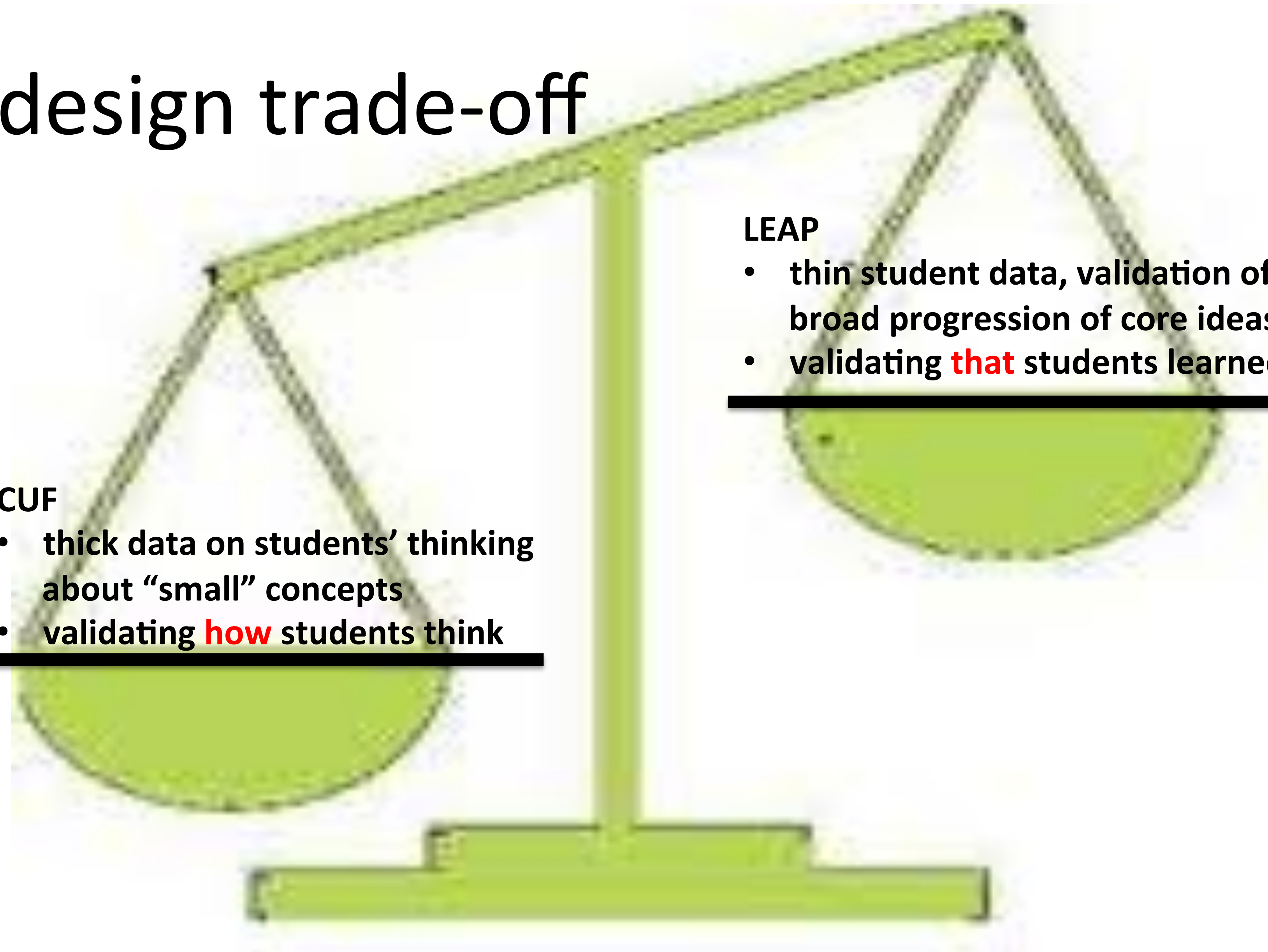
- Developed an empirically-based instructional sequence on functions;
- We are "validating" hypothesized learning trajectories by
  - Implemented the sequence through 16 lessons in an 8-week classroom teaching experiment at each of grades K-2 and a series of individual teaching experiments (pre-mid-post); conducted ongoing analysis re students' understanding to begin to identify levels of thinking and refine subsequent lessons; all classroom observations and interviews videotaped
  - Identifying profiles in students' thinking ("levels of sophistication"), as a result of our instructional sequence, through qualitative analysis of individual and classroom teaching experiments.

Challenge 2: Grain size ⟶ design trade-offs

CUF:
- Brief time span (8 weeks)
- Narrow content focus
- Focus on validating trajectories in thinking
- “Learning Trajectories”

LEAP:
- Broad time span (grades 3-7)
- Broad content focus (all of early algebra)
- Focus on validating instructional sequence
- “Learning Progressions”

design trade-off



LEAP

- **thin student data, validation of broad progression of core ideas**
- **validating that students learned**

CUF

- **thick data on students' thinking about "small" concepts**
- **validating how students think**

We can't do both at the same time, but they both have value.

Challenge 3: The "starting point" matters…and can drive what gets foregrounded

Starting Point:

- the project's goals
  - can dictate grain size and methodology
- whether you start with a "stable" curriculum

LEAP: A Design Challenge

- Starting Point: Develop an empirically-based "new" instructional sequence for grades 3-7 by pulling together multiple domains of early algebra (vs. relatively known sequence on functions in CUF)

This (and our project goal) foregrounded our focus on

“Was it a reasonable sequence?”

This (and our project goal) foregrounded our focus on

"Was it a reasonable sequence?"

(and backgrounded our focus on progressions in thinking)

Inevitable design trade-offs: Different grain sizes and starting points can significantly impact the product we create:

- broad treatment of big ideas might produce validated curricular progressions, whereas
- narrow treatment of small ideas might produce validated progressions in thinking (and might more carefully script sequences/learning in ways that support individual classroom practice)

Can these co-exist and how do we get the best of both worlds?

Challenge 4: The starting point* matters…and can determine the nature of trajectories in thinking

*Where you start with what children know

E.g., the nature of children's thinking about functions in grade 3 (Project LEAP) vs. grades K-1 (CUF)

Similar tasks & sequence

Similar trajectories in thinking

Similar tasks & sequence

Significantly different trajectories in thinking

Children in Grades K-1…

- Children did not have a pre-existing fixed focus on recursive (scalar) relationships that needed to be unpacked and seemed to more naturally focus on two co-varying quantities rather than one;

- Children did not have strong aversions to or misconceptions about using a variable to represent an unknown quantity;

- Children were more likely to represent a function rule as an equation (e.g., $R + R = V$) rather than as an expression;

- Children seemed more at ease using symbolic notation and natural language;

- Children did not exhibit object/quantity confusion with variables.

Are trajectories in thinking so tightly connected to the instructional sequence and the knowledge students start with so as to challenge their "validity in use" of LTs/LPs?

- If the same sequence/tasks can lead to significantly different trajectories depending on the audience (grade K vs grade 3), how does this impact validity in use?

Some things we still don’t know

- How “big” is a big idea? What is the grain size for the concepts on which we focus? When is a concept (or trajectory) too small?
- Should instructional sequences in early algebra be organized around content or algebraic thinking practices?
- What do we do with all of our LTs/LPs so that they have validity in use?

Arrgghhhhh!