EXPLORING K-2 STUDENT UNDERSTANDINGS OF VISUAL REPRESENTATIONS IN ALGEBRAIC REASONING

Tufts UNIVERSITY
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PIs: Bárbara M. Brizuela & Maria Blanton

Project Team: Susanne Strachota, Angela Murphy Gardiner, Mary Caddle, Sophia Raymond, Alina Kilcullen, & Charlotte Ferguson

The following research questions [**RQ**] guided our work:

RQ1: What are trajectories of learning in how grades K–2 children understand visual representations such as tables, graphs, and diagrams of algebraic relationships?
RQ2: What features of tasks or instruction facilitate movement in students' thinking within the trajectories?
RQ3: What are similarities and differences in how children understand visual representations of algebraic relationships across the content dimensions of functional thinking and generalized arithmetic?

WHAT ARE WE LEARNING?

Based on prior work we assumed students would construct and interpret tables prior to constructing and interpreting graphs. However, through the classroom teaching experiments (CTEs) we observed students:

- *simultaneously* constructing and interpreting both tables and graphs
- unexpected strategies for graphing, such as lines between the quantities on the *x*- and *y*-axis

Both of these surprises are small slices of the progression of learning to graph. In our next steps, we will dig deeper into better understanding how students simultaneously construct and interpret tables and graphs, and the strategies that are most effective for teaching them to do this.

CONCEPTUAL FRAMEWORK

Early Algebra Conceptual Framework: We use Kaput's (2008) conceptual analysis of algebra to frame early algebraic thinking around four fundamental practices: (1) generalizing mathematical relationships and structure; (2) representing generalized relationships in diverse ways; (3) reasoning with generalized relationships; and (4) justifying generalizations (Blanton et al., 2011; Kaput, 2008). **Our primary focus in this work is on the representing practice**, with a secondary focus on the remaining three practices, given the interconnections among the four practices.

We build on Selling's (2016) framework of how students use representations. From lesser to greater sophistication, students shift

1. *from using a single type of representation* (e.g., tables)
2. *to using different types of representations* (e.g., tables and graphs)
3. *to using multiple representations for the same concept* (e.g., tables, graphs, and equations)
4. and eventually connecting different representations of the same concept.

These frameworks guided the design of early algebra curricular progression that supports students in representing algebraic content using tables, graphs, and diagrams. As students move through this curricular progression, we engaged them in *creating* representations, *communicating* about representations, and *reasoning* with representations in the context of the aforementioned content dimensions with an aim to support them in developing representational fluency (Sandoval et al., 2000; Suh et al., 2008; Zbiek et al., 2007) about tables, graphs, and diagrams and to understand how they engage in that process.

EMERGING ANALYSIS

Using a grounded approach (Strauss & Corbin, 1990) we are beginning our analysis with open coding. Here we report on some initial observations from one student's first (pre) and third (post) interviews. We include a QR code to view initial observations from the second (mid) interview. We observed this student move from learning to *interpret* tables and graphs to *constructing and reasoning* with tables and graphs.

METHOD

This Spring 2023 (up until May 2023) we conducted CTEs in Kindergarten, Grade 1, and Grade 2 at an elementary in the Northeastern U.S. We taught 16 lessons in Kindergarten and 14 lessons in each of Grades 1 and 2. Lessons were taught by a teacher-researcher and were about 30-40 minutes. All lessons were video recorded and are currently being transcribed. Additionally, we interviewed four students from each of the three classrooms before, during, and after the intervention. Scan the QR code for a summary of the lesson sequence.

EMERGING FINDINGS

When asked about the relationship between the number of birds and number of bird wings, Luca, a first grade student, started by interpreting and adding some values to a table in his first interview. In the second and third interviews, he began to construct and reason with his own table and graph. Due to space limitations we only share data from the first and third interview.

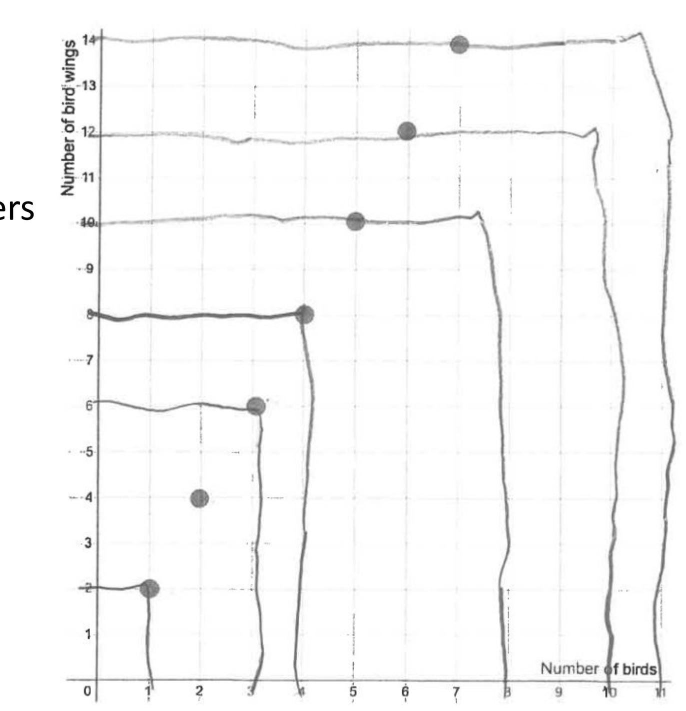
PRE-INTERVIEW STUDENT WORK

"One bird has two wings. Two birds have four wings. Three bird have six wings. Four birds have eight wings."

This response exhibits particular functional thinking (Blanton et al., 2015).

Number of birds	Number of bird wings
1	2
2	4
3	6
4	8
5	10
10	20

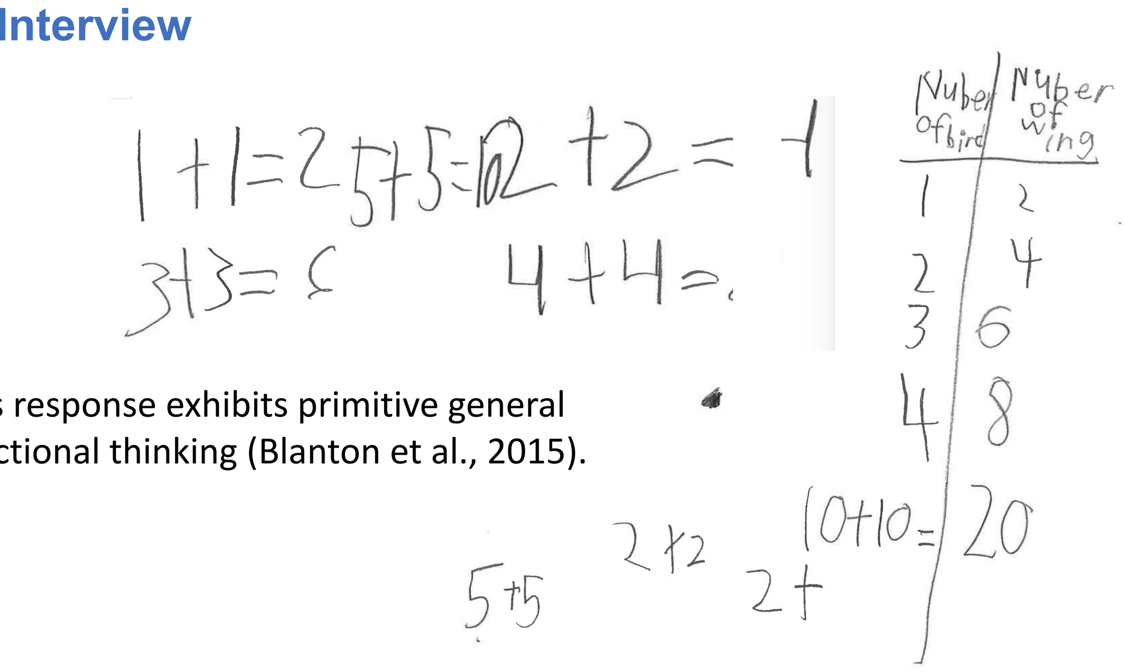
"Eight wings is four birds…11 birds have 14 wings (draws lines connecting these numbers on his graph)."



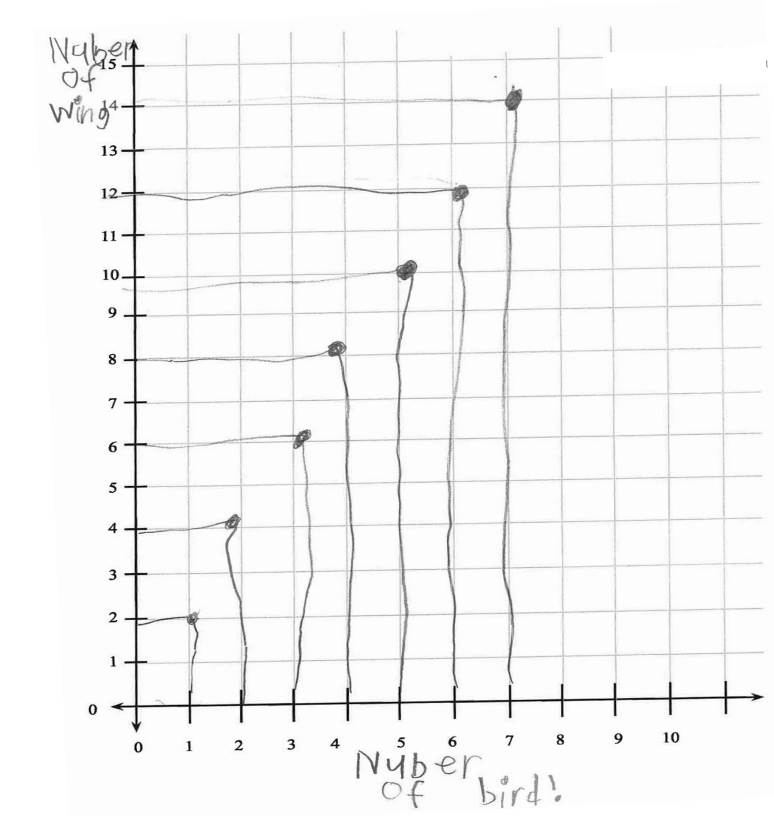
SCAN ME Luca's 2nd Interview

POST-INTERVIEW STUDENT WORK

"Two birds have four wings (uses pencil to show point and lines connecting 2 birds and 4 wings)."



This response exhibits primitive general functional thinking (Blanton et al., 2015).



When asked to show the number of wings for ten birds, Luca gestures with his pencil up from ten birds and indicates where the point (10, 20) would be.

SCAN ME for references

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